Activism and Indexing in Equilibrium

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Abstract

We study a dynamic general equilibrium production economy intermediated by actively and passively managed funds. Two actively managed types arise endogenously through a search mechanism. Activists are socially valuable because they improve productive efficiency. Quants displace activists by buying efficient firms. A representative household allocates its wealth to activist, quant, and index funds. We characterize the equilibrium impact of changes in search costs, productive efficiency, and index fees. While a decrease in the passive index fee is beneficial to households through cheaper diversification, it can negatively impact the overall efficiency of the economy by reducing the number of activists.

Keywords: Activism, delegated fund management, index funds

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1 Introduction

If a firm’s managers are destroying shareholder value, an asset manager faces two primary choices: voice or exit. Voice, or activism, is generally understood to describe a broad range of shareholder actions designed to influence a firm’s real or financial policy decisions. As such, it is recognized as one important component of corporate governance. We build a general equilibrium model of the asset management industry with activism at its heart. Our goal is to understand how activism coexists with alternative asset management channels, with a particular emphasis on the implications of fee compression for household welfare and for the relative sizes of the different fund channels.

The foundation of the model is a set of productive technologies representing firms. A representative household can invest in firms through three different types of intermediaries: a passive index, activists, and quants. The passive fund allows the representative household access to an equal-weighted portfolio of all firms at low or zero fees. Its existence captures the intuition that the household does not require a skilled manager to spread capital evenly across firms, but the household does required a skilled manager to identify the most efficient firms.

An activist creates a fund by incurring a search cost to match with a firm, exercising voice, and forcing the firm to use its capital efficiently. The firm remains efficient as long as it is matched with an activist. This is the direct benefit of voice. However, the activist manager also generates a positive externality because the increase in firm efficiency contributes to increased performance for the passive index. A quant manager can identify and invest in an efficient firm, but the quant does not exercise voice. The arrival of the quant destroys the match between an activist and a firm, and the resulting fund may operate less efficiently than an activist fund and with different fees. In the absence of a match with an activist, a firm held by the quant will revert over time, at an exogenous rate, to an inefficient use of its
capital. Both activists and quants choose fees optimally in a symmetric Nash equilibrium, given the state of competition in the managed fund market.

The household determines the demand for each of the three types of funds by solving an optimal consumption and portfolio choice problem. In equilibrium, the markets for goods and fund shares clear, determining the aggregate size of each fund sector. We perform a number of numerical experiments that examine the key factors determining the interactions among agents. In particular, how do search costs and fee structures affect the relative sizes of the fund management sectors, and what can we learn about the relationship between active management and household welfare?

We demonstrate that, for a wide range of parameter values, a non-degenerate stationary distribution for each fund type exists in the model’s steady state. Our baseline parameterization also demonstrates how fund market competition impacts assets under management for the household through diversification and spillover effects. Although the equilibrium characteristics of activists and quants are symmetric in many regards, their responses to some changes to equilibrium parameters are not necessarily identical. There is a “predator/prey” component to the dynamics of the relative sizes of the activist and quant sectors that lead to interesting interaction effects.

For example, as the number of activist funds grows, the household invests more in the sector overall as it becomes better diversified. However, more managed funds also make the passive index more efficient, leading to a spillover effect that eventually makes assets under management in the active sector fall. Quants also generate a spillover effect. If more of them are in the market, activist assets under management fall even sooner. This dynamic between activists and quants also has a differential impact on the present value of fees collected. When quant-owned firms are less efficient than activist-owned firms, quants charge lower fees to households. However, these lower fees are not necessarily beneficial to households due to a reduction in the quality of the household investment opportunity set.
The dynamics of the model are particularly sensitive to variations in the cost of intermediation. If quant search costs fall too much, say through better technology to detect efficient firms, the size of the activist sector can crash ultimately leading to a crash in the quant sector too. This renders the market less efficient and leads to lower household welfare. Across the board increases in the cost of intermediation through a fee on the passive index lead to an increase in fees charged by activists and quants. This has a direct negative impact on household welfare, but this increase in fees can also adversely impact the composition of activists relative to quants again leading to lower household welfare through a reduction in the quality of the investment opportunity set.

While declining passive index fees and fee compression have become commonplace in the asset management industry\(^1\) understanding their effect on investor welfare requires a dynamic assessment of their impact on the incentives for activists to participate in the asset management industry. A decrease in the passive index fee provides the benefit of cheaper diversification options, but it can be harmful to overall efficiency if the reduction in fees seriously curtails the number of activists willing to participate in the fund management industry, as we document in a simple example.

In focusing on the role of voice in equilibrium, we have largely abstracted from the conventional focus of mutual fund and hedge fund research: does active management deliver alpha through the selection of mispriced securities? We say “largely” because quant managers do engage in a form of identification of mispriced assets. Our purpose is not to argue that the conventional focus is unimportant but rather to argue that activism may play a more important role in understanding asset management in general equilibrium than has generally been acknowledged. This view is consistent with the recent survey evidence in McCahery, Sautner, and Starks (2016) and the empirical literature on activism summarized in Brav,  

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\(^1\)See for example Morningstar’s Annual Fund Fee Study available at [https://www.morningstar.com/lp/annual-us-fund-fee-study](https://www.morningstar.com/lp/annual-us-fund-fee-study)
The rest of the paper is organized as follows: After placing our work in the context of the relevant literature on both shareholder monitoring and active management, we present the model and characterize the equilibrium. We then examine how the equilibrium responds to changes in the cost of intermediation along several dimensions. The final section of the paper concludes. All proofs are in the appendix.

2 Our Contribution Relative to the Existing Literature

In order for any shareholder (or group of shareholders) to effectively monitor firm managers, they must solve a free-rider problem; i.e., the costly efforts of shareholders who monitor the firm also benefit other shareholders who exert no effort. This problem might be overcome by large shareholders via takeovers, as in Shleifer and Vishny (1986). Yet ownership that is too highly concentrated could lead to sub-optimally tight control by shareholders (Burkart, Gromb, and Panunzi, 1997), or to weaker monitoring incentives stemming from reduced liquidity and price informativeness (Holmström and Tirole, 1993). Assuming that the right number of shareholders are paying the right amount of attention, providing improved incentives to the firm’s managers remains a nontrivial problem (Core, Guay, and Larcker, 2003).

The results in Admati, Pfleiderer, and Zechner (1994) provide theoretical arguments against the use of voice by demonstrating that the equilibrium level of monitoring is well below the socially optimal level of monitoring, and DeMarzo and Urosevic (2006) extend this result to show that, over time, an activist will actually hold a perfectly diversified portfolio. Marinovic and Varas (2019) extend these findings by showing that whether or not the activist monitors and whether or not that monitoring increases firm value depends critically on the
presence of information asymmetry about the activist’s ability.$^2$

Despite these substantial incentive and information obstacles, McCahery, Sautner, and Starks (2016) present survey evidence that over half of institutional investors engage directly with the management and boards of the firms in which they hold stock, presumably to the benefit of shareholders at large. The survey article of Brav, Jiang, and Kim (2015a) provides additional empirical evidence of the impact of activism on firms. In particular, Brav, Jiang, and Kim (2015b) show that activism empirically has a positive impact on plant-level productivity.

We sidestep the discordance of the theoretical results with the empirical findings by abstracting from the specific details of how activists change firm policies – and by extension firm value. Instead, we assume a matching technology that pairs fund managers (both activists and quants) to firms. Once the match is formed the monitoring requires no costly effort by managers, and the representative household has perfect information about which firms are matched to each type of manager. We use voice and monitoring as a metaphor for how active managers can deliver value to households in equilibrium in the absence of asymmetric information. We are interested in understanding the factors that affect the relative sizes of the active and passive sectors of the asset management industry and how the structure of the asset management industry affects household welfare in general equilibrium under perfect information.

Pastor and Stambaugh (2012) provide an alternative explanation for the relative sizes of the active and passive sectors of the fund industry. Their focus is on explaining the poor performance of active funds relative to a passive benchmark (the “active management puzzle”) in an equilibrium model where funds face decreasing returns to scale; i.e., when

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$^2$This additional layer of delegation also introduces new agency problems that have been the focus of a large literature on optimal managerial contracting; see, for example, the theoretical work of Bhattacharya and Pfleiderer (1985), Stoughton (1993), Heinkel and Stoughton (1994), Starks (1987), and the empirical work in Almazan et al. (2004).
fund performance erodes because too many managers trade in related strategies. Their explanation of active vs. passive sector sizes is complementary to ours. Their model is more general than ours in the sense that “alpha” in their analysis might come from a variety of manager actions whereas we focus exclusively on voice. Our model is more general than theirs in that we focus on a production economy and our household explicitly solves a multiperiod (infinite-horizon) portfolio problem. Finally, in [Pastor and Stambaugh (2012)], investors learn about managerial skill while our framework assumes complete information.

Whereas we focus on voice as a source of positive spillovers from active management, Buss and Sundaresan (2020) focus on improved price informativeness as a source of positive spillovers from active management. In theoretical and empirical examples, they document a surprising positive relationship between high passive ownership and price informativeness, as high passive ownership increases the incentive for information production by active managers. In our setting, passive and active fund shares are dynamic and endogenously determined, so the relationship is nuanced. When passive ownership is high because index funds charge low fees, activists and quants compete by reducing fees too, which can benefit households overall despite reduced monitoring of firms by activists.

In contrast to our work that studies the equilibrium dynamics of the size of the actively managed and passively managed sectors, works such as Gärleanu and Pedersen (2018, 2019) and Corum, Malenko, and Malenko (2020) study the impact on market efficiency of endogenous costly information acquisition in a noisy rational expectations equilibrium with one period of trade. Gärleanu and Pedersen (2018) explore how market efficiency is impacted when investors incur search costs to find asset managers. Building on this work, Gärleanu and Pedersen (2019) focus on the impact on passive investing in a setting with a fixed number of active asset managers. Corum, Malenko, and Malenko (2020) build on both of these works by adding a governance choice to both active and passive managers in a risk neutral investor setting. They show that increased passive investing can have an ambiguous impact
on the level of governance. Given we assume long-lived risk averse households as well as entry and exit of fund managers, we are able to explore fund management sector dynamics and household welfare that are driven by spillover and diversification effects.

We assume that passive managers provide diversification services at low cost, and they do not engage in any use of voice or (by definition) other active strategies. Azar, Schmalz, and Tecu (2018) argue that even passive managers can facilitate collusion in concentrated industries, to the detriment of households. However, there is a growing literature that contradicts the findings in Azar, Schmalz, and Tecu (2018); see, for example Dennis, Gerardi, and Schenone (2020).

3 Model

We study a continuous-time, infinite-horizon economy in which the fund management industry intermediates between households and productive investment opportunities. We build the model from the bottom up, beginning with the productive technologies, or firms. Investment managers who are either activists or quants search for opportunities to match with firms, thereby forming funds. A lower cost passive index fund also exists which invests equally in all firms. Finally, a representative household optimally allocates capital across funds. Having closed the model, we solve and analyze fund and capital market dynamics in general equilibrium.

3.1 Firms

Capital may be productively invested in \( N \) firms. All firms have identical productivity of capital \( \mu \), such that a firm \( j \) with capital \( K_{j,t} \) produces output at gross rate \( \mu K_{j,t} dt \). Capital
in firm $j$ also depreciates at a rate

$$K_{j,t} \left[ -\delta_{j,t} dt + \bar{\sigma} d\bar{W}_t + \sigma dW_{j,t} \right].$$

(1)

The Brownian motion $\bar{W}_t$ captures a capital depreciation shock common to all firms, whereas the Brownian motion $W_{j,t}$ captures firm-specific depreciation, which is independent of $\bar{W}_t$ or $W_{i,t}$, $i \neq j$. At any given time, the deterministic depreciation rate $\delta_{j,t}$ takes one of three values. When a firm is in an activist fund, depreciation takes its lowest value $\delta_{j,t} = \delta_A$, reflecting efficiency gains from monitoring. When a firm previously monitored by an activist becomes part of a quant fund, depreciation becomes $\delta_{j,t} = \delta_Q \geq \delta_A$, allowing for an immediate drop in efficiency following cessation of activist scrutiny. A firm that is not part of any actively managed fund is assumed to be inefficiently run, with $\delta_{j,t} = \delta_U > \delta_Q$. This simple reduced-form specification captures the idea that active management produces economic benefits through monitoring, particularly in the case of activists.

### 3.2 Funds

Activists seek out inefficient firms to monitor, thereby rendering them efficient. Firms are efficient as long as they are combined with an activist. How these managers effect change or “voice” is modeled in reduced form. However, we have in mind mechanisms for voice as discussed in the survey evidence in McCahery, Sautner, and Starks (2016). We also assume that activism is always successful. If an activist chooses to invest in a firm, that firm is always efficiently run as modeled by the lower depreciation rate $\delta_{j,t} = \delta_A$.

In contrast, quantitative fund managers, or quants, are able to invest in efficient firms that are currently monitored by activists. Quant managers are the manifestation, in the

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3As noted earlier, reality dictates that activism will not always be successful and can be curtailed by free rider problems (Grossman and Hart 1980; Shleifer and Vishny 1986) or liquidity issues (Coffee 1991). Recent empirical work on activism includes Becht et al. (2017) and Boyson and Pichler (2018).
model, of the free-rider problem of Shleifer and Vishny (1986). Quants perform little or no monitoring themselves. Instead, they displace incumbent activists, which may immediately lead to a partial loss of efficiency: $\delta_{j,t} = \delta_Q \geq \delta_A$. In addition, once the activist stops monitoring the firm, there is a chance that it reverts to full inefficiency $\delta_{j,t} = \delta_U > \delta_Q$. If and when this occurs, the quant fund dissolves.

A large but finite number of ex-ante identically skilled managers search for opportunities in the labor market for asset managers. Managers match with firms to form investment funds, which may be either quant funds or activist funds. A fund consists of one firm and one manager. An unemployed (potential) manager may search for opportunities as either an activist or a quant, or may choose not to search at all. The choice is sensitive to the current competitiveness of the fund market. At any moment, there are $n_t$ incumbent activist funds and $m_t$ incumbent quant funds. Potential managers who choose not to search can be thought of as remaining in the general household pool, earning reservation utility with certainty equivalent value $cK_t$, where $K_t$ is the aggregate capital stock, or equivalently aggregate wealth.$^4$

If a potential manager chooses to search, he pays a flow cost $\zeta_A K_t dt$ while searching for activist opportunities or $\zeta_Q K_t dt$ for quant opportunities. New matches are formed at rates

$$\hat{n}_t^{1-\nu} (N - n_t - m_t)\nu, \quad \hat{m}_t^{1-\nu} n_t^\nu,$$

for activists and quants, respectively, where $\hat{n}_t$ is the number of potential managers searching for activist opportunities and $\hat{m}_t$ the number of potential quants searching. Although empirical work on fund manager labor market dynamics is comparatively scant, the constant returns to scale matching technology is standard in the broader labor literature; see for example Petrongolo and Pissarides (2001), Rogerson, Shimer, and Wright (2005), and

$^4$The number of potential managers is not generally important to the setup, but we assume that there are a large number of potential managers relative to firms.
In our experiments, we assume that the elasticity parameter $\nu = 1/2$ and that there is no difference in $\nu$ between activists and quants. This corresponds to a prior belief that matching is equally difficult for both manager types.

Constant returns to scale implies that individual potential activist and quant managers find matches at the respective rates

$$
\left( \frac{N - n_t - m_t}{\hat{n}_t} \right)^\nu, \quad \left( \frac{n_t}{\hat{m}_t} \right)^\nu.
$$

That is, the chance a searching manager succeeds depends on the tightness of the labor market, a ratio of available target firms to searching managers. For most of our analysis, the rate at which new quant funds are formed is also the rate at which existing activist funds dissolve. Finally, quant funds dissolve at an exogenous rate $\theta_Q$ individually, or $\theta_Q m_t$ in the aggregate. When a quant fund dissolves, an efficient firm reverts to being inefficient.

Potential managers take their earnings present value as given when making search decisions. An incumbent activist manager earns fee income with time $t$ present value $\Phi(n_t, m_t, K_t)$. An incumbent quant manager has present value of expected earnings $\Psi(n_t, m_t, K_t)$. Fee income depends on funds under management, which reflects household asset allocation, for which we later solve in equilibrium.

Potential managers will enter the labor market (search) until the net present value of a new fund is zero. For activists, $\hat{n}_t$ is the number of searchers such that

$$
(\Phi(n_t + 1, m_t, K_t) - \zeta K_t) \left( \frac{N - n_t - m_t}{\hat{n}_t} \right)^\nu = \zeta_A K_t.
$$

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5 We allow for the possibility that some activists also exit for exogenous reasons, at rate $\theta_A$ individually or $\theta_A n_t$ in the aggregate. We set $\theta_A = 0$ in our baseline parameters.

6 We allow $\hat{n}_t$ to take nonnegative real values, rather than restricting it to nonnegative integer values, hence Equation (4) holds with equality. $\hat{n}_t$, and $\hat{m}_t$ in Equation (5), can be interpreted more generally as search intensities. As a practical matter, continuously valued search intensities facilitate convergence of the numerical solution routine.
When a potential activist matches with a target firm, he exchanges fee revenues for his household consumption stream. The exchange must be favorable enough, and the intensity of matching high enough, to justify the search costs. The equivalent condition for quants is

\[(\Psi(n_t + 1, m_t, K_t) - \zeta K_t) \left(\frac{n_t}{m_t}\right)^\nu = \zeta_Q K_t.\]  

(5)

We verify that one solution to the activist and quant fee-setting problems has the property that the present value of fees is homogeneous of degree one in capital \(K_t\), such that they can be written \(\Phi(n_t, m_t, K_t) = \phi(n_t, m_t)K_t\) for activists, and \(\Psi(n_t, m_t, K_t) = \psi(n_t, m_t)K_t\) for quants. This allows us to solve for \(\hat{n}_t\), the number of potential managers seeking to start activist funds, such that

\[\hat{n}_t = \left(\frac{\phi(n_t + 1, m_t) - \zeta}{\zeta_A}\right)^{\frac{1}{\nu}} (N - n_t - m_t),\]  

(6)

whereas \(\hat{m}_t\), the number of potential managers searching to be quants, satisfies

\[\hat{m}_t = \left(\frac{\psi(n_t, m_t + 1) - \zeta}{\zeta_Q}\right)^{\frac{1}{\nu}} n_t.\]  

(7)

Finally, a third type of investment fund exists alongside quants and activists: a passive index fund, which invests equally in each of the \(N\) firms. This fund is unique, requires no manager, and charges a small, exogenous, and constant fee rate \(\bar{\pi}\) proportional to capital under management.\(^7\) In addition to capturing competition from low-cost index funds, the passive fund reflects the idea that households do not require skilled managers in order to spread capital evenly across all firms, but they do require skilled managers to identify the most efficient firms. In a simple way, this captures the idea that fund managers are better informed than households.

\(^7\)In our baseline parameters, it is zero.
3.3 Households

A representative household allocates capital to investment funds in order to maximize expected lifetime utility. While the state of competition among funds matters to households, there is no reason for households to differentiate between funds of a given type. For example, each activist charges an identical fee proportional to capital, and each invests in a unique — but equivalent — efficient firm. Hence it is optimal, for purposes of diversification, for the household to invest the same amount in each activist fund. The relevant question is how much the household should invest in activist funds collectively.

Without loss of generality, let the first $n_t$ firms be activist funds, the next $m_t$ firms be quant funds, and the remaining $N - n_t - m_t$ firms belong exclusively to the the passive index. We define composite Brownian motions

$$W_{A,t} = \sum_{j=1}^{n_t} W_{j,t}, \quad W_{Q,t} = \sum_{j=n_t+1}^{n_t+m_t} W_{j,t}, \quad W_{I,t} = \sum_{j=n_t+m_t+1}^{N} W_{j,t},$$

(8)

representing risk specific to activists, quants, and the passive fund, respectively.

While we have in mind that the passive fund fee $\bar{\pi}$ is small, a fraction $\frac{N-n_t-m_t}{N}$ of its holdings are inefficient, with higher depreciation rate $\delta_U$. Both activists and quants invest exclusively in relatively efficient firms, with deterministic capital depreciation $\delta_A$ for activists and $\delta_Q$ for quants. However these funds also charge commensurately higher fees proportional to capital under management. Fee rates may vary depending on the amount of competition in the fund market. The quant’s fee rate is $\bar{\psi}_t > 0$, and the activist’s fee rate is $\bar{\phi}_t > 0$. Households and potential managers take these fees as given. Later we explain how incumbent managers strategically set fees to maximize revenues.

Households allocate aggregate capital, $K_t$, across the three types of funds, and also choose

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8Recall that funds consist of non-overlapping firm-manager pairs but the passive index invests equally in every firm.
the consumption-capital ratio $c_t$. Let $a_t$ be the activists’ fraction of capital, $q_t$ the quants’ fraction of capital, and the residual $1 - a_t - q_t$ is invested in the passive index. The aggregate capital accumulation process is

$$\frac{dK_t}{K_t} = (\mu - c_t)dt + \sigma dW_t$$

$$+ \begin{bmatrix} a_t \\ q_t \\ (1 - a_t - q_t) \end{bmatrix} \begin{bmatrix} - (\bar{\phi}_t + \delta_A)dt + \frac{\sigma}{\sqrt{m_t}} dW_{A,t} \\ - (\bar{\psi}_t + \delta_Q)dt + \frac{\sigma}{\sqrt{m_t}} dW_{Q,t} \\ - (\bar{\pi}_t + \delta_{I,t})dt + \frac{\sigma}{\sqrt{N}} \left(\sqrt{N - n_t - m_t} dW_{I,t} + \sqrt{n_t} dW_{A,t} + \sqrt{m_t} dW_{Q,t}\right) \end{bmatrix},$$

where

$$\delta_{I,t} = \frac{1}{N} \left((N - n_t - m_t)\delta_U + n_t\delta_A + m_t\delta_Q\right)$$

is the mean depreciation rate of firms in the index.

Households have CRRA utility, and choose consumption and capital allocation to maximize their lifetime expected utility,

$$\max_{\{c_t, a_t, q_t\}_{t=0}^{\infty}} E_0 \left[ \int_0^\infty e^{-\rho t} u(c_t K_t) dt \right],$$

subject to Equation (9).

### 4 Equilibrium

The economic state is summarized by the tuple $\omega_t = (m_t, n_t)$, a Markov process capturing current competition in the fund markets based on the number of quants and activists, as well as the aggregate capital stock $K_t$. Since the economy is time-homogeneous, we omit time subscripts in the solution.
4.1 The Fund Managers’ Problems

We begin by solving for a manager’s entry decision taking fees as given. Expectation under the risk-neutral measure is denoted $E^Q$, and the instantaneous risk free discount rate is $r(w)$, i.e., a function of the Markov state only. Define the space of Markov states $\Omega$, and for current state $\omega \in \Omega$, the instantaneous rate of transition to $\omega' \in \Omega$ is $\lambda_{\omega,\omega'}$, and $\Lambda$ the transition matrix, with $\lambda_{\omega,\omega'} = -\sum_{\omega'' \in \Omega, \omega'' \neq \omega} \lambda_{\omega,\omega''}$. The equilibrium fee and a manager’s entry decision are determined simultaneously, but we discuss them sequentially.

The instantaneous hazard rate $\beta_A(\omega)$, serves two distinct purposes in the model. First, it captures the possibility that a particular activist is driven out of the market. This occurs when the activist’s firm becomes part of a quant fund in our baseline example, or because an activist exits for exogenous reasons in one of our extensions. Second, the hazard rate also limits the conditions under which potential activists will search to form new funds. It is possible for the present value of activist fees to be positive even if the current flow of fees is non-positive, i.e., if the household would choose a zero or short position in activist funds. To prevent this, we impose that activists immediately exit if their current fee flow turns negative. It follows that

$$\beta_A(\omega) = \begin{cases} \frac{\lambda_{(m,n),(m+1,n-1)}}{n} + \theta_A & \text{if } a(\omega) > 0, \\ \infty & \text{otherwise.} \end{cases} \tag{12}$$

The present value of an incumbent activist’s fees is

$$\Phi(\omega_t, K_t) = E^Q_t \left[ \int_t^\infty e^{\int_t^s \beta_A(\omega_v) + r(\omega_v) dv} \tilde{\phi}(\omega_s) a(\omega_s) n_s K_s ds \right]. \tag{13}$$
Similarly, for quants we define

$$\beta_Q(\omega) = \begin{cases} 
\theta_Q & \text{if } q(\omega) > 0, \\
\infty & \text{otherwise}, 
\end{cases}$$

and the present value of an incumbent quant’s fees is

$$\Psi(\omega_t, K_t) = \mathbb{E}_t^Q \left[ \int_t^\infty e^{\int_t^s \beta_Q(\omega_v) + r(\omega_v) dv} \bar{\psi}(\omega) q(\omega_s) \frac{ds}{m_s} K_s ds \right].$$

Using a generalized Feynman-Kac theorem, the stochastic integrals in Equation (13) and Equation (15) can be expressed as the solutions to the following partial differential equations (PDEs):

$$\mathcal{A} \Phi(\omega, K) - (\beta_A(\omega) + r(\omega)) \Phi(\omega, K) + \frac{\phi(\omega) a(\omega)}{n} K = 0$$

and

$$\mathcal{A} \Psi(\omega, K) - (\beta_Q(\omega) + r(\omega)) \Psi(\omega, K) + \frac{\psi(\omega) q(\omega)}{m} K = 0,$$

where $\mathcal{A}$ denotes the infinitesimal generator for $(\omega_t, K_t)$. The following proposition characterizes the managerial fees:

**Proposition 1.** *Under the assumption that $\Phi(\omega, K) = \phi(\omega) K$ and $\Psi(\omega, K) = \psi(\omega) K$, the PDEs in Equation (16) and Equation (17) can be written as*

$$\mathcal{A} \Phi(\omega, K) = \phi(\omega) (r(\omega) - c(\omega)) K + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'} [\phi(\omega') - \phi(\omega)] K,$$

and

$$\mathcal{A} \Psi(\omega, K) = \psi(\omega) (r(\omega) - c(\omega)) K + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'} [\psi(\omega') - \psi(\omega)] K.$$
The instantaneous transition rates defining fee dynamics are

\[
\lambda_{(m,n),(m,n+1)} = \hat{n}^{1-\nu}(N - n - m)^\nu, \quad (20) \\
\lambda_{(m,n),(m+1,n-1)} = \hat{m}^{1-\nu}n^\nu, \quad (21) \\
\lambda_{(m,n),(m-1,n)} = \theta_A n, \quad (22) \\
\lambda_{(m,n),(m,n-1)} = \theta_Q m, \quad (23)
\]

and zero to all other states, subject to the additional restriction that transition intensity to states outside of \{0 \ldots N\} is always zero.

Given the entry decision, we now endogenize how fee rates are set by incumbent managers. We assume managers behave strategically, albeit in a limited sense. For each state \(\omega\), we solve for a symmetric Nash equilibrium in which each manager chooses his fee rate to maximize his flow of revenues, i.e., the product of his fee rate and the household’s allocation to his fund. In doing so the manager takes into account the response of the household to a potential change in fee rate, given the fees charged by the manager’s competitors. However incumbent managers do not consider how their fee strategy alters fund market dynamics through its effects on the labor market search behavior of potential managers. This assumption is primarily for tractability, since it decouples the determination of fee rates from equilibrium fund market dynamics. However it seems reasonable that while fund managers might respond to current competitive pressures, they may not anticipate how their behavior will alter the evolution of the fund market in the future.

**Proposition 2.** Given a state \(\omega = (m,n)\), let \(\bar{\phi}(\omega)\) denote the optimal fee rate for each of the \(n\) revenue maximizing activist funds and \(\tilde{\psi}(\omega)\) be the optimal fee rate for each of the \(m\)
revenue maximizing quant funds. The optimal managerial fees are:

\[
\tilde{\phi}(\omega) = \frac{(2(N - n) - m + 1)(N(\delta_I(\omega) + \bar{\pi} - \delta_A) - m(\delta_Q - \delta_A))}{(2N - m - n + 1)(2(N - m - n) + 1)} - \frac{m(N(\delta_I(\omega) + \bar{\pi} - \delta_Q) - n(\delta_A - \delta_Q))}{(2N - m - n + 1)(2(N - m - n) + 1)},
\]

\[
\tilde{\psi}(\omega) = \frac{(2(N - m) - n + 1)(N(\delta_I(\omega) + \bar{\pi} - \delta_Q) - n(\delta_A - \delta_Q))}{(2N - m - n + 1)(2(N - m - n) + 1)} - \frac{n(N(\delta_I(\omega) + \bar{\pi} - \delta_A) - m(\delta_Q - \delta_A))}{(2N - m - n + 1)(2(N - m - n) + 1)}.
\]

In the special case where activist and quant firms are equally efficient ($\delta_A = \delta_Q$), activist and quant funds will optimally charge identical rates. In this case, equilibrium fee rates reduce to

\[
\tilde{\phi}(\omega) = \tilde{\psi}(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi} - \delta_A)}{2N - m - n + 1}.
\]

We focus on this case in our numerical examples.

### 4.2 The Household’s Problem

The household’s problem, in Equation (11), is essentially a conventional planner’s problem with affine production technologies and no capital adjustment costs. The novelty is that the risk and productivity characteristics of the production technologies arise from equilibrium in the fund manager labor market.\footnote{Our solution of the household’s problem follows Sotomayor and Cadenillas (2009), who establish optimality conditions in a setting similar to Merton (1969), but with market opportunities and utility contingent upon a continuous-time Markov process. See in particular Theorem 3.2, which covers the case where utility is not directly contingent on the Markov state. Although we generalize to a production setting and endogenous investment opportunities, the household’s problem remains very similar to Sotomayor and Cadenillas (2009).}

We make the usual conjecture that the value function can be written

\[
V(\omega, K) = \frac{1}{1 - \gamma} H(\omega) K^{1-\gamma},
\]

for a function $H(\omega)$ to be determined.
The value function satisfies Hamilton-Jacobi-Bellman (HJB) equation

\[
\frac{1}{1-\gamma} \left[ \rho H(\omega) - \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'} H(\omega') \right]
\]

\[
= \max_{c,a,q} \left\{ u(c) + (\mu - c - a(\bar{d}(\omega) + \delta_A) - q(\bar{p}(\omega) + \delta_Q) - (1 - a - q)(\bar{p} + \delta_I(\omega)))H(\omega) \right. \\
- \frac{\gamma H(\omega)}{2} \left[ \sigma^2 + \frac{\sigma^2}{N} \left( 1 - 2aq + \left( \frac{N}{n} - 1 \right) a^2 + \left( \frac{N}{m} - 1 \right) q^2 \right) \right] \right\}.
\]

The solution to the asset allocation problem is similar to Merton (1969): it reflects the mean returns and covariance matrix for the funds. Because we assume frictionless capital allocation and time-additively-separable utility, state transitions do not introduce a hedging component to the asset allocation problem. Optimal \( c, a, \) and \( q \) are

\[
c(\omega) = H(\omega)^{-1/\gamma},
\]

\[
a(\omega) = \frac{n ( (N - m)(\delta_I(\omega) + \bar{p} - \bar{d}(\omega) - \delta_A) + m(\delta_I(\omega) + \bar{p} - \bar{p}(\omega) - \delta_Q))}{\gamma \sigma^2 (N - m - n)},
\]

\[
q(\omega) = \frac{m ( (N - n)(\delta_I(\omega) + \bar{p} - \bar{q}(\omega) - \delta_Q) + n(\delta_I(\omega) + \bar{p} - \bar{d}(\omega) - \delta_A))}{\gamma \sigma^2 (N - m - n)}.
\]

To condense notation, write the drift and variance of the household’s optimal portfolio returns, respectively, as

\[
\hat{\mu}(\omega) = \mu - a(\omega)(\bar{d}(\omega) + \delta_A) - q(\omega)(\bar{p}(\omega) + \delta_Q) - (1 - a(\omega) - q(\omega))(\delta_I(\omega) + \bar{p}),
\]

\[
\hat{\sigma}(\omega) = \sigma + \frac{\sigma^2}{N} \left( 1 - 2a(\omega)q(\omega) + \left( \frac{N}{n} - 1 \right) a(\omega)^2 + \left( \frac{N}{m} - 1 \right) q(\omega)^2 \right).
\]

Substituting the optimal portfolio choice and consumption and making use of the condensed notation, the HJB equation is

\[
\gamma H(\omega)^{(\gamma-1)/\gamma} + \left( 1 - \gamma \right) \left( \frac{\hat{\mu}(\omega) - \frac{\gamma \hat{\sigma}(\omega)}{2}}{\rho} \right) H(\omega) + \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'} H(\omega') = 0.
\]
The risk-free rate under the household’s pricing kernel is

\[ r_f(\omega) = \rho + \gamma(\bar{\mu}(\omega) - c(\omega)) - \frac{1}{2}\gamma(1 + \gamma)\sigma^2(\omega). \] (35)

5 Results

We characterize model behavior via a series of propositions and numerical examples. We begin with some analytical comparative static results conditional on the state of the managed fund market \( \omega \). These provide intuition for some mechanisms at work in the model, and inform our numerical experiments. We then characterize the dynamic model for a set of baseline parameters. Finally we study the phenomenon of fee compression, and related changes in the relative competitiveness of index funds, activists, and quants.

5.1 Conditional Comparative Statics

We begin by establishing some properties of fees when quants run their firms less efficiently than activists, i.e., \( \delta_Q > \delta_A \). We show that this is a necessary condition for quants to undercut activist fees in our model. We further show that transitions to states in which quants replace activists are not desirable to the household when quants charge lower fees, because the portfolio opportunity set worsens when activists are replaced by comparatively inefficient quants. It is for this reason, in addition to parsimony, that we later focus on numerical examples in which \( \delta_Q = \delta_A \): this allows the greatest scope for quants to have a positive impact on household welfare in the dynamic equilibrium, not by undercutting activists on fees, but by potentially increasing the overall state of competition in the managed fund market.\(^{10}\)

The results in this section condition on the state of fund market competition, \( \omega = \)

\(^{10}\)We explore dynamic equilibria with \( \delta_A \neq \delta_Q \) in Section 5.4
(m, n). They reflect what would occur in a static version of our model, or one in which state transitions were exogenous and unanticipated.

**Proposition 3.** For any state $\omega$ such that $n + m < N$, quants charge lower fees than activists, $\bar{\psi}(\omega) < \bar{\phi}(\omega)$, if and only if quant firms are run less efficiently than activist firms, $\delta_Q > \delta_A$.

Although the details are specific to our setting, Proposition 3 makes a general point: Absent frictions or behavioral biases, quants offering a substitute product for “traditional” actively managed funds should not find it optimal to charge relatively lower fees unless their product is inferior in some way. Hence if we observe quant funds offering their strategies at a discount, it is unlikely that the quant fund possesses an advantage in efficiency — more likely the opposite.¹¹ Our assumption that the difference in efficiency arises at the firm level (due to inferior monitoring) rather than at the fund level (due to, e.g., higher overhead) is important for the aggregate productivity of capital and expected returns to the passive index, but not for the results on relative fees.

We can also relate efficiency, fees, and expected returns conditional on state $\omega$.

**Proposition 4.** Suppose quant firms are less efficient than activist firms, $\delta_Q > \delta_A$. Then for any state $\omega$ such that $n + m < N$, quant funds have lower net expected returns than activist funds: $\mu - \delta_Q - \bar{\psi}(\omega) < \mu - \delta_A - \phi(\omega)$.

Although Proposition 4 conditions on our choice of parameters $\delta_A$ and $\delta_Q$, it would be equivalent to condition on relative fees, per Proposition 3. In combination, these propositions imply that any discount in fees charged by quants relative to activists does not fully offset the relative inefficiency of the firms managed by quants. It could be detrimental to households

¹¹There are, of course, other potential explanations not present in our model: perhaps the household requires time to learn that the quantitative strategy offers equivalent returns, for example, a form of information friction from which we abstract. We also rule out sophisticated intertemporal strategic behavior by incumbent funds, oriented to affect entry and exit dynamics.
if “low cost” quants replaced activists, even if they charged lower fees, because expected returns for quants net of fees are still lower than for activists.

The following proposition shows that this is indeed the case.

**Proposition 5.** Assume $\delta_Q > \delta_A$, and consider a transition from state $\omega = (m, n)$ to state $\omega' = (m + \Delta m, n - \Delta m)$, for some integer $\Delta m > 0$, and $m + n < N$. This transition replaces $\Delta m$ activists with quants. The transition increases activist fees, increases quant fees, and decreases average firm efficiency: $\bar{\phi}(\omega') > \bar{\phi}(\omega)$, $\bar{\psi}(\omega') > \bar{\psi}(\omega)$, and $\delta_I(\omega') > \delta_I(\omega)$. The household’s portfolio opportunity set worsens when quants replace activists if quants are less efficient than activists.

Although the above result may seem obvious in light of the quants’ relative inefficiency, when considered in terms of fees it is less intuitive: quants charge lower fees than activists, yet transitioning to a state in which quants replace activists will increase fees for both types, and will immediately make the household worse off.

Because of the results above, we focus on numerical examples with $\delta_A = \delta_Q$, which implies that there is no immediate loss in efficiency when a quant displaces an activist. Although it is not optimal for quants to undercut activist fees in such examples, we believe these parameters offer quants the best chance to provide social value, perhaps by increasing the overall level of competition (number of funds) in the managed fund market.

In addition to competing with each other, activists and quants also compete with the passive index fund. In contrast to activist and quant fees, the passive index fee $\bar{\pi}$ is exogenous and constant. Although modeled in reduced form, we have in mind a fully competitive market for identical index funds, such that $\bar{\pi}$ represents labor and capital costs entailed to run an index fund at zero profit. In keeping with this interpretation, a reduction in $\bar{\pi}$ can be interpreted as a form of increased efficiency. The following proposition summarizes its effects.
Proposition 6. For any state $\omega$, decreasing the passive index fee $\bar{\pi}$ decreases both the activist fee $\bar{\phi}(\omega)$ and the quant fee $\bar{\psi}(\omega)$. Therefore a reduction in $\bar{\pi}$ improves the household’s portfolio opportunity set conditional on state $\omega$.

Because we assume time-additively-separable household utility, Proposition 6 suggests that a reduction in index fees $\bar{\pi}$ is conditionally welfare improving for households, in the sense that expected utility within a given state $\omega$ increases due to the improvement in the household’s portfolio opportunity set. Although it is still possible that a reduction in $\bar{\pi}$ could unconditionally reduce household welfare, such a result must hinge on the dynamic effects of the fee reduction, i.e., on the incentives of activists and quants to search in their respective labor markets. We present relevant examples in Section 5.3.

5.2 Dynamic Characterization for Baseline Parameters

We characterize the dynamic model via numerical examples. Our objective is to describe the main mechanisms driving the equilibrium state transition process, by explaining how households and fund managers view opportunities across states. In our model, potential fund managers consider two key elements that vary with the model state: the present value of their fees if they successfully form a fund, and the supply of firms suitable for inclusion in a fund. To understand the present value of fees, we must see how households allocate investment in each state, and how incumbent funds set their fees. We cover these topics in turn after briefly describing baseline parameters and the resulting stationary distribution of quants and activists.

Our baseline parameters in Table 1 are chosen to be consistent with observed economic quantities. It is worth noting a few of the parameters. First, we assume that quant and activist firms are equally efficient, with $\delta_Q = \delta_A = 0$. Therefore they offer equivalent products, and charge identical fees. By contrast inefficient firms have a depreciation of $\delta_U = 4\%$. 

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Given the mean productivity of capital $\mu$ is 11%, the efficiency gain from monitoring is economically meaningful. Second, our number of firms, $N = 25$, is chosen for fast equilibrium computational time. Experiments with larger numbers of firms lead to similar results. Finally, the search parameters, while consistent with the labor literature, are largely ad hoc in a managed money setting. Our baseline is to start with identical search parameters for both activists and quants.

Figure 1 illustrates the stationary distribution of activists and quants under the Table 1 parameters. The top panels summarize the joint density, while the bottom panels summarize the marginal densities. In equilibrium, a non-degenerate distribution exists where the modal percentage of firms managed by activists is 36% and the modal percentage of firms managed by quants is 28%. In separate experiments, the stationary distribution for activists and quants is non-degenerate across a wide range of parameters. Given that fees are endogenous, it is optimal for activists and quants to set fees such that they remain in the fund management sector.

While Figure 1 conveys information about the number of activists and quants in equilibrium, Figure 2 demonstrates how fund market competition impacts assets under management (AUM) through the household’s portfolio problem. The left panel plots AUM as a fraction of wealth across the three fund types as a function of fund market competition, as summarized by the number of activists ($n$) and quants ($m$). The right panel shows different slices of the surface conditional on 1, 5, or 10 quants.

Figure 2 is a good place to understand how competition among funds has diversification and spillover effects. As the number of activist funds in the market increases from, for example, a single fund, the household will initially invest more in the activist sector overall.

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12 Again, see Petrongolo and Pissarides (2001), Rogerson, Shimer, and Wright (2005), and Petrosky-Nadeau, Zhang, and Kuehn (2018) for examples.

13 The pictures produced by slices through the quant and activist distributions are symmetric. Households are indifferent between holding similarly sized activist and quant sectors because they both provide the same expected returns at the same fee.
because it becomes better diversified. But more managed funds also implies more efficient firms in the passive index, a spillover effect that eventually causes AUM among activists to decline with the entry of additional activists. Since quant funds also produce the spillover effect, the point at which AUM begins to decline with additional activists comes sooner when there are many quants in the market. Whatever funds are not allocated to activists or quants are invested in the passive index, so passive AUM is the mirror image of active sector investment. Passive investment peaks when actively managed funds are few and poorly diversified, or many and generating a large spillover.

Based on the logic of the previous paragraph, one would expect states with more activists \((n)\) and quants \((m)\) to be better for households, because the managed fund sector is better diversified and the firms in the passive index are on average more efficient. Although this reasoning is essentially correct, it omits the effects of fund manager labor market dynamics on conditional household welfare. Figure 3 shows household welfare conditional on state \(\omega = (m, n)\), normalizing the capital stock to unity, i.e., \(\frac{1}{1-\gamma}H(\omega)\). This measure takes account of probable changes to the household’s portfolio opportunity set.

Although conditional welfare increases with \(m\) and \(n\) independently at the margins, households would prefer a high number of activists and a low number of quants from a welfare perspective. This reflects state transition dynamics in combination with our assumption of finite firms. When the fund market is saturated with many activists but few quants, then most firms are efficiently run, and they are likely to remain efficient for a long time. This is because the most likely change in the fund sector is that a quant replaces an activist, and our baseline parameters assume that quants run their firms as efficiently as activists. If instead the fund market is saturated with many quants and few activists, then most firms are efficiently run at the moment, but they are likely to become less efficient in the future.

\(^{14}\)The plots for some activist and quant combinations are truncated due to that combination having zero probability in the steady state distribution. For example, for \(m = 10\) quants, the maximal number of activists that still has a positive probability is \(n = 13\).
That is because the most likely change in the fund management sector is that a quant exits when its firm reverts to inefficiency.

Therefore a result similar to Proposition 3 holds even when quants and activists are equally efficient ($\delta_Q = \delta_A$), once state transitions are taken into account: household welfare decreases when quants replace activists.

To understand the incentives of potential managers to search for activist or quant jobs, we show conditional fees in Figure 4. The top panel of Figure 4 plots the fee rate, common for both activists and quants, for different amounts of fund competition. Equilibrium fees range between slightly above 0 basis points and slightly below 200 basis points. Increased competition, either through more activists or quants, lowers the fee rate.

However potential managers are forward looking, in the sense that they consider the conditional present value of fees. These are shown in the lower two panels of Figure 4. On a conditional basis, the present value of fees typically decreases with the number of activists and quants in the market. Even though AUM in the sector as a whole may increase due to improved diversification, as previously illustrated in Figure 2, AUM per fund generally decreases as the number of competing funds increases. In combination with the decrease in fee rates due to increased competition, this implies fee flows that decrease with the number of activists and quants in the market. The effect of such declining fee flows is mitigated somewhat by the possibility of transitioning to a state with reduced competition, which becomes increasingly likely as the number of incumbent activists and quants grows large.

The other component of potential managers’ search decisions is the state of firms in the market: potential activists match with inefficient firms, whereas potential quants match with efficient firms currently run by activists. All else equal, the likelihood that an activist’s search is successful is highest when few firms are efficient and $n + m$ is low, whereas the likelihood that a quant’s search is successful is highest when many firms are monitored by activists and $n$ is high.
These considerations about search success interact with the present values of fees, shown in Figure 4 to determine search intensity, shown in Figure 5. For activists, search intensity is highest when there are few activists or quants, because the present value of fees is high, and inefficient target firms are plentiful. Intensity declines sharply as \( n \) and \( m \) increase. For quants, search intensity is more diffuse, peaking in the interior of the state space when there are sufficiently many activists to target but not so many that fees are low. Quant search intensity is declining in the number of incumbent quants, since these represent competition but not targets.

Per Equation (2), search intensity and the availability of target firms determines the rate at which new funds enter and, in the case of activists, exit. In turn this determines the stationary distribution we began with in Figure 1.

Having established its key mechanisms, we use our model to shed light on topics of practical importance and current interest. First we consider fee compression — the reduction in active management fees coincident with increased passive management — and its implications for households and the managed fund industry. Related to this topic, we present comparative statics for index fees. Finally we consider changes in the relative competitiveness of quants and activists.

5.3 Fee Compression and Indexing

As documented in Duval (2020), the past two decades saw trends of increasing index share of AUM and declining actively managed fund fees, even as the number of actively managed funds continued to grow. Between 2000 and 2019, average equity mutual fund fees declined from 1.06% to 0.74%, index share grew from 7.5% to 24.2% of long term mutual fund AUM, and the number of mutual funds increased from roughly 7,000 to over 21,000.\(^{15}\) The

\(^{15}\)The number of mutual funds is from the ICI Factbook, available online at [https://www.icifactbook.org/ch2/20_fb_ch2](https://www.icifactbook.org/ch2/20_fb_ch2), which includes all mutual funds. According to Duval (2020) there were 271 index funds in 2000 versus 492 index funds in 2019, implying that the number of funds excluding index funds has
phenomenon of decreasing fees in combination with increased competition and a shift towards passive management is referred to as fee compression.

Our dynamic model implies substantial variation in managed fund fees, even if there are no structural changes to the market, as illustrated in Figure 6. For our baseline parameters, fund fees range from about 0.1% to 1.9% of AUM, with most of the probability mass on fees between 0.5% and 1.5%. The top panel of Figure 6 shows that fees decline with the number of managed funds in the market, as competition becomes more intense. But it also shows that fractional AUM for the managed fund sector as a whole (i.e., quants and activists combined) is non-monotonic in fees. As fees decline from their maximum, AUM initially increases, peaking with fees of about 1%, then AUM declines as fees fall towards their minimum. Finally, the bottom right panel of Figure 6 shows that the market share of quants also peaks when the managed fund industry as a whole is large, and shrinks far more than activist market share when fees become small.

Although we do not deny the existence of structural changes or secular trends in the managed fund industry, the general pattern of falling fund fees, increasing numbers of funds, declining managed fund share of AUM and increasing passive fund share of AUM is consistent with our dynamic model absent structural (or parameter) changes, even without calibrating to match these observations. As shown in Figure 6, the modal state is approximately peak active management with fees of around 1%. From that modal state, a realized decline in fees of 1/4% and attendant increase in passive AUM of 5% or more is not improbable.

The previous analysis assumes passive index fees are constant, however they have in reality declined in recent years, from an average of 0.27% in 2000 to 0.07% in 2019, according to Duval (2020). Since our model does not incorporate endogenous time-variation in index approximately tripled. However the number of actively managed funds is not separately stated, and the total includes some funds of funds that may not be considered active.

16 Available data from industry reports does not address the relative impact of fee compression on quants versus activists or fundamental managers.
fees, we present a comparative static analysis of index fees, in Figure 7. Here our results average over possible states \( \omega \) according to the stationary distribution implied by each set of model parameters. We vary index fees from 0% to 1.5%, a wider range than seen empirically, to illustrate the full range of model outcomes.

The bottom left panel of Figure 7 shows the relationship between the exogenous index fee and the endogenous mean actively managed fund fee, which we place on the horizontal axis for consistency with Figure 6. Mean actively managed fund fees decline from 2% to 1% as index fees fall. Mean actively managed fund AUM falls from approximately 100% to 50% as the initially uncompetitive index fund gains ground, and the mean number of actively managed funds falls from almost 18 to about 16. Similar to Figure 6, activists fare better in low fee scenarios than do quants. Unlike in Figure 6, all relationships are monotonic.

Based on our model it seems plausible that a decline in index fees contributed to a decline in actively managed fund fees and an increase in passive index market share. However it is interesting that the mean impact of declining index fees on managed fund fees is dwarfed by typical model dynamics without structural parameter changes, at least for the 0.2% decline in index fees observed since the year 2000.

So far the unconditional comparative static results from our dynamic model are consistent with the conditional results in Proposition 6: a decrease in the index fee decreases activist and quant fees. The bottom left panel of Figure 8 shows that welfare implications are also consistent with Proposition 6: lower index fees increase household welfare unconditionally as well as conditionally, with parameters otherwise unchanged from Table 1.

Although higher index fees are typically bad for households, there are counterexamples. Low index fees make diversified investment inexpensive, but this crowds outs active management, which reduces average firm efficiency. If the reduction in efficiency is large enough relative to the risk reduction from improved diversification, household welfare declines. In our baseline calibration, the reduction in efficiency is mitigated because relatively more quants
are driven out of the market. This is because potential quants face a reduction in target firms currently monitored by activists, in addition to a reduction in AUM from the shift towards passive investment. As previously discussed in the context of Figure 3, households prefer activists to quants, even when activist and quant firms are as efficiently run, because firms run by activists remain efficient for longer than those run by quants. The relative shift towards activists is one reason that the reduction in index fees is beneficial for households on net.

The right panel of Figure 8 shows an alternative calibration in which quants are shut down, and repeats the unconditional comparative static analysis with index fees ranging from 0% to 1.5%, using the stationary distributions corresponding to each set of model parameters. In combination with these changes, low index fees are bad for households. The shift towards passive investment deters activists from entering the market, and the reduction in average firm efficiency is greater than the value of the fee reductions. In this way the effects of cheap indexing on the incentive to engage in activism can overturn the intuition from Proposition 3 which holds fund market composition constant.

5.4 Changes in Activist or Quant Competitiveness

The previous section considered increased competition for active fund managers in general, in the form of decreased index fees. However recent advances in computing technology, e.g., developments in machine learning combined with improved data aggregation and integration, may have enabled easier identification of investment prospects by active funds, potentially increasing competitiveness. Additionally, the Volcker Rule enacted in 2014 as part of the Dodd-Frank Wall Street Reform and Consumer Protection Act curtailed proprietary trading inside banks, impacting the labor market for active fund managers. We model these changes

17Baseline parameters in Table 1 are modified to have fund manager reservation utility $c = 0.08\%$, exogenous quant exit rate $\theta_Q = 1$, and exogenous activist exit rate $\theta_A = 0.2$. 

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in reduced form by varying the search costs for potential managers to form funds.

Although our first motivating example of improved computing technology applies most naturally to quants, active managers of all types make use of it to some extent. The Volcker Rule example potentially impacted the labor market for both types of managers. The effects of varying activist or quant search costs are shown in the left and right panels of Figure 9, respectively. We vary search costs from 0.0001 to 0.002, recalling that costs are 0.001 for activists and quants in the baseline example. Unconditional means average over possible states \( \omega \) according to the stationary distribution implied by each set of model parameters, with all parameters except search costs per Table 1.

In most respects reducing activist search costs does as one would expect. More potential managers search for activist opportunities when costs are low, so the mean number of activists increases, and mean fees decline. Some of the increase in activist funds comes at the expense of quants, which decline in mean number with lower activist search costs. The total number of funds increases, so household welfare increases with lower activist search costs, as firms are more efficiently run on average. Somewhat counterintuitive is the response of mean activist AUM to search costs. Activists AUM initially increases as costs decline and more activists enter, but AUM declines for very low search costs, as the spillover effect from increased activism becomes so strong that households shift towards passive investment, around \( \zeta_A = 0.0005 \). In this sense decreased activist search costs may be bad for activists.

The effects of declining quant search costs are given in the right panel of Figure 9. There are some similarities with the previous case: unless quant search costs become very low, declining search costs increase the mean number of quants and quant AUM. However, the number of activists declines rapidly because new quants displace activists, so the total number of funds declines and fund fees increase with falling quant search costs. Consequently household welfare decreases with falling quant search costs. Perhaps most interesting, sufficiently low quant search costs can cause a collapse in the fund industry, as so few activists
remain that quants have difficulty finding efficient firms in which to invest, and the total number of funds crashes, causing household welfare to crash also.

Over time it is also plausible that the incentives for funds to monitor the companies in which they invest have shifted, or that the cost of monitoring has changed. We model these changes in reduced form by varying the depreciation rate associated with quant or activist firms. This alters the household’s portfolio opportunity set directly, by changing the quality of individual quant or activist funds, in addition to altering the composition of the portfolio opportunity set, by changing the equilibrium composition of funds. Although we style activists as the initial reformers of inefficient firms, in reality some quant funds do exercise voice too, and the degree to which they do so could be captured by changes in $\delta_Q$.

The effects of varying activist or quant depreciation are shown in the left and right panels of Figure 10, respectively. Unconditional means average over possible states $\omega$ according to the stationary distribution implied by each set of model parameters, with all parameters except search costs per Table 1. Unlike our previous examples after Section 5.1, activist and quant fees differ in Figure 10 except when $\delta_A = \delta_Q = 0$, matching our baseline parameter values. As activist depreciation declines from 0.005 towards zero, in the left panel, the number of activists increases, activist AUM increases, and activist fees increase slightly. The total number of funds increases also, so household welfare increases as firms become more efficient on average. Activist gains in AUM come at the expense of the passive index as much as from quants, so total fund AUM increases as activist depreciation decreases. The biggest impact on quants is a decline in fees, which fall from 1.2% to 1% as activist depreciation falls. Interestingly mean fund fees fall also, as increased competition lowers fees even though funds offer a more attractive product on average. The overall impression is that the fund industry is healthier and more competitive when activists run their funds more efficiently.

As shown in the right panel of Figure 10, the effects of decreasing quant depreciation are less salubrious. In short, changes in market composition dominate changes in the quality of
quant funds. Each quant fund runs its firm more efficiently with lower depreciation, but this causes households to reallocate AUM from activists to quants. In turn this induces more quants to enter, displacing activists. Because activists run their firms more efficiently than quants in this example (except when quant depreciation is zero), and activist firms remain efficient for longer than quant firms on average, household welfare declines as $\delta_Q$ falls. The total number of funds declines also, and fees increase. With $\delta_A = \delta_Q = 0$, at the left extreme of the horizontal axes, there are fewer funds competing but each is run very efficiently, which supports high fees. Interestingly mean total fund AUM increases as $\delta_Q$ falls, despite rising fees and falling numbers of funds, suggesting that it is better for the fund industry than it is for households when quants run their firms efficiently.

6 Conclusion

Activist fund management has been transformed over the past two decades with both significant changes in the costs of running a fund, with the rise in competition from index products, delivered through both open-end funds and ETFs, and competition from managers building new forms of quantitative strategies. At the same time, following the financial crisis, there has also been an argument in the political arena about the size of the financial service sector as a whole.

In order to better understand the competitive pressures between these different types of funds and their impact on household welfare, we construct a stylized general equilibrium model of the fund industry. In our model, activist managers have social value by exercising voice to improve the operating efficiency of the assets that they own. This does not preclude the possibility that managers deliver value through the identification of mispriced securities, but we do not need to assume mis-pricing skill for managers’ services to improve household welfare.
Although the model is simple enough to clearly delineate the important margins affecting the decisions of managers and households, it is rich enough to deliver nontrivial implications for the impacts of fees and costs on the composition of the fund industry and household welfare. The model can deliver both endogenous fee structures and non-degenerate steady state distributions of the sizes of the different sectors. Through simple quantitative experiments, we use the model to understand the impact on the fund industry and household welfare of changes in search costs, managerial efficiency, and fee compression. We highlight interesting endogenous nonlinearities in the model equilibrium and provide some insight into the spillover effects created by active management.

We do not view our results as the final word on fund industry dynamics and household welfare in general equilibrium. However, by providing a fully articulated example of the importance of general equilibrium considerations, we hope that it will stimulate further research in this area.
References


Appendix

Proof of Proposition 1. We use a generalized Feynman-Kac theorem to express the value of the stochastic integral above as the solution to a PDE, which reduces to a simple system of equations in the Markov state variable due to homogeneity of the valuation in $K$.\footnote{See Zhu, Yin, and Baran (2015), Theorem 3.2.} Let $\mathcal{A}$ be the infinitesimal generator for $(\omega_t, K_t)$. Then the function $\Phi$ satisfies

$$\mathcal{A} \Phi(\omega, K) - (\beta_A(\omega) + r(\omega)) \Phi(\omega, K) + \frac{\bar{\phi}(\omega)a(\omega)}{n} K = 0. \quad (36)$$

Recall our supposition $\Phi(\omega, K) = \phi(\omega) K$. Note that $\Phi_{KK} = 0$ and the drift of $dK_t$ is $(r(\omega_t) - c(\omega_t)) K_t$ under the risk-neutral measure, where $c(\omega_t)$ is the aggregate consumption-capital ratio, interpretable as a dividend yield. Following equation (2.9) in Zhu, Yin, and Baran (2015), we have

$$\mathcal{A} \Phi(\omega, K) = \phi(\omega)(r(\omega) - c(\omega)) K + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'}[\phi(\omega') - \phi(\omega)] K. \quad (37)$$

The optimal $\phi(\omega)$ solves

$$\phi(\omega)(r(\omega) - c(\omega)) K + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'}[\phi(\omega') - \phi(\omega)] K \quad (38)$$

$$- (\beta_A(\omega) + r(\omega)) \phi(\omega) K + \frac{\bar{\phi}(\omega)a(\omega)}{n} K = 0,$$

$$\Rightarrow \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'}[\phi(\omega') - \phi(\omega)] - (\beta_A(\omega) + c(\omega)) \phi(\omega) + \frac{\bar{\phi}(\omega)a(\omega)}{n} = 0. \quad (39)$$

Similarly, the function $\Psi$ satisfies

$$\mathcal{A} \Psi(\omega, K) - (\beta_Q(\omega) + r(\omega)) \Psi(\omega, K) + \frac{\bar{\psi}(\omega)q(\omega)}{m} K = 0, \quad (40)$$

where

$$\mathcal{A} \Psi(\omega, K) = \psi(\omega)(r(\omega) - c(\omega)) K + \sum_{\omega' \in \Omega} \lambda_{\omega, \omega'}[\psi(\omega') - \psi(\omega)] K. \quad (41)$$

After simplification this leaves

$$\sum_{\omega' \in \Omega} \lambda_{\omega, \omega'}[\psi(\omega') - \psi(\omega)] - (\beta_Q(\omega) + c(\omega)) \psi(\omega) + \frac{\bar{\psi}(\omega)q(\omega)}{m} = 0. \quad (42)$$

Subject to $\phi(\omega)$ and $\psi(\omega)$ satisfying Equation (39) and Equation (42), respectively, manage-
rial search intensities \( \hat{n} \) and \( \hat{m} \) satisfy Equation (6) and Equation (7), respectively. Therefore instantaneous transition rates are

\[
\lambda_{(m,n),(m,n+1)} = \hat{n}^{1-\nu}(N-n-m)^\nu, \\
\lambda_{(m,n),(m+1,n-1)} = \hat{m}^{1-\nu}n^\nu, \\
\lambda_{(m,n),(m,n-1)} = \theta_A n, \\
\lambda_{(m,n),(m-1,n)} = \theta_Q m,
\]

and zero to all other states, subject to the additional restriction that transition intensity to states outside of \( \{0 \ldots N\} \) is always zero.

Proof of Proposition 2. For some state \( \omega = (m,n) \) with \( n > 0 \), consider an individual activist who chooses his fee rate \( \bar{\phi}'(\omega) \), while the remaining \( n-1 \) activists charge fee \( \bar{\phi}(\omega) \) and \( m \) quants charge fee \( \bar{\psi}(\omega) \). Given his choice of fee, the individual activist will attract capital

\[
a'(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi}) - (N-m-n+1)(\bar{\phi}'(\omega) + \delta_A) - m(\bar{\psi}(\omega) + \delta_Q) - (n-1)(\bar{\phi}(\omega) + \delta_A)}{\gamma \sigma^2 (N-m-n)}.
\]

The individual activist solves revenue maximization problem

\[
\max_{\bar{\phi}'(\omega)} \bar{\phi}'(\omega)a'(\omega),
\]

which has solution

\[
\bar{\phi}'(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi} - \delta_A) - m(\bar{\psi}(\omega) + \delta_Q - \delta_A) - (n-1)\bar{\phi}(\omega)}{2(N-m-n+1)}.
\]

Similarly, for some state \( \omega = (m,n) \) with \( m > 0 \), an individual quant choosing fee rate \( \bar{\psi}'(\omega) \), while \( n \) activists charge fee \( \bar{\phi}(\omega) \) and the remaining \( m-1 \) quants charge fee \( \bar{\psi}(\omega) \), attracts capital

\[
q'(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi}) - (N-n-m+1)(\bar{\psi}'(\omega) + \delta_Q) - n(\bar{\phi}(\omega) + \delta_A) - (m-1)(\bar{\psi}(\omega) + \delta_Q)}{\gamma \sigma^2 (N-m-n)}.
\]

The individual quant solves revenue maximization problem

\[
\max_{\bar{\psi}'(\omega)} \bar{\psi}'(\omega)q'(\omega),
\]

which has solution

\[
\bar{\psi}'(\omega) = \frac{N(\delta_I(\omega) + \bar{\pi} - \delta_Q) - n(\bar{\phi}(\omega) + \delta_A - \delta_Q) - (m-1)\bar{\psi}(\omega)}{2(N-n-m+1)}.
\]

The symmetric Nash equilibrium is one in which all activists choose identical state-
contingent fee $\bar{\phi}(\omega)$, all quants choose identical state-contingent fee $\bar{\psi}(\omega)$, and no individual fund manager has an incentive to deviate from the rate schedule.

Proof of Proposition 3. From Equations (24) and (25),

$$\bar{\phi}(\omega) - \bar{\psi}(\omega) = \frac{(N - n - m)(\delta_Q - \delta_A)}{2(N - m - n) + 1}.$$ 

Since $2(N - m - n) + 1 > 0$ always and $N - n - m > 0$ by assumption, the sign of the difference in fees is given by $\delta_Q - \delta_A$.

Proof of Proposition 4. Since $\delta_Q > \delta_A$ and $n + m < N$, the result follows immediately because

$$\bar{\phi}(\omega) - \bar{\psi}(\omega) = \frac{N - n - m}{2(N - m - n) + 1}(\delta_Q - \delta_A) < \delta_Q - \delta_A.$$ 

Proof of Proposition 5. After simplification,

$$\bar{\phi}(\omega') - \bar{\phi}(\omega) = \bar{\psi}(\omega') - \bar{\psi}(\omega) = \frac{(N - m - n)\Delta m(\delta_Q - \delta_A)}{(2N - m - n + 1)(2(N - m - n) + 1)} \geq 0,$$

with strict inequality when $m + n < N$, and

$$\delta_f(\omega') - \delta_f(\omega) = \frac{\Delta m(\delta_Q - \delta_A)}{N} > 0.$$ 

Proof of Proposition 6. This simply requires examination of the partial derivatives of optimal fees with respect to $\bar{\pi}$:

$$\frac{\partial \bar{\phi}}{\partial \bar{\pi}} = \frac{\partial \bar{\psi}}{\partial \bar{\pi}} = \frac{(2(N - m - n) + 1)N}{(2N - m - n + 1)(2(N - m - n) + 1)} > 0.$$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean productivity of capital</td>
<td>$\mu$ 0.11</td>
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<tr>
<td>Inefficient deterministic cap. depreciation</td>
<td>$\delta_U$ 0.04</td>
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<tr>
<td>Quant deterministic cap. depreciation</td>
<td>$\delta_Q$ 0.00</td>
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<tr>
<td>Activist deterministic cap. depreciation</td>
<td>$\delta_A$ 0.00</td>
</tr>
<tr>
<td>Firm-specific std. dev. of cap. depreciation</td>
<td>$\sigma$ 0.3</td>
</tr>
<tr>
<td>Systematic std. dev. of cap. depreciation</td>
<td>$\bar{\sigma}$ 0.13</td>
</tr>
<tr>
<td>Number of firms</td>
<td>$N$ 25</td>
</tr>
<tr>
<td>Household relative risk aversion</td>
<td>$\gamma$ 4</td>
</tr>
<tr>
<td>Household subjective discount rate</td>
<td>$\rho$ 0.02</td>
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<tr>
<td>Fund labor market share coefficient</td>
<td>$\nu$ 0.5</td>
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<tr>
<td>Activist exogenous exit rate</td>
<td>$\theta_A$ 0</td>
</tr>
<tr>
<td>Quant exogenous exit rate</td>
<td>$\theta_Q$ 0.35</td>
</tr>
<tr>
<td>Activist search cost</td>
<td>$\zeta_A$ 0.001</td>
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<tr>
<td>Quant search cost</td>
<td>$\zeta_Q$ 0.001</td>
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<tr>
<td>Passive index fee</td>
<td>$\pi$ 0</td>
</tr>
<tr>
<td>Manager reservation utility coeff.</td>
<td>$\zeta$ 0</td>
</tr>
</tbody>
</table>

Table 1: **Parameter values.** The table reports the baseline parameter values used in our numerical examples.
Figure 1: Stationary Distribution of Activists and Quants: Base Parameters. The top left figure shows the joint stationary density of the number of quant ($m$) and activist ($n$) funds in the market, under example parameter values in Table 1. The top right figure represents the same information as a contour plot. The bottom figures show the marginal stationary densities of the number of activist ($n$) and quant ($m$) funds in the market.
Figure 2: AUM: Base Parameters. The figure shows assets under management (AUM) in each of the three fund sectors, as a fraction of aggregate wealth, conditional on the state of competition in the fund manager market. Fund market competition is summarized by the tuple \((m, n)\), where \(m\) is the number of quant funds, and \(n\) the number of activist funds. The top panel summarizes total activist AUM, the middle panel summarizes total quant AUM, and the bottom panel summarizes passive AUM. Left panels show three dimensional surfaces, whereas right panels show two dimensional slices conditional on the number of quants \(m\). Parameter values are per Table I.
Figure 3: Household Welfare: Base Parameters. The figure shows properties of household welfare conditional on the state of competition in the fund market. Fund market competition is summarized by the tuple \((m, n)\), where \(n\) is the number of activist funds, and \(m\) the number of quant funds. The left panel plots relative to the number of activists \(n\), while the right panel plots relative to the number of quants \(m\). Parameter values are per Table 1.
Figure 4: Fees: Base Parameters. The figure shows properties of fees conditional on the state of competition in the fund market. Fund market competition is summarized by the tuple \((m, n)\), where \(n\) is the number of activist funds, and \(m\) the number of quant funds. The top panels summarize the fee rate for both activists and quants, the middle panels summarize the present value of activist fees per fund, and the bottom panels summarize the present value of quant fees per fund. The left panels plot relative to the number of activists \(n\), while the right panels plot relative to the number of quants \(m\). Parameter values are per Table 1.
Figure 5: Potential Manager Search Intensity: Base Parameters. The figure shows the search intensity of potential activists (top panel) and potential quants (bottom panel), conditional on the state of competition in the fund market. Fund market competition is summarized by the tuple \((m, n)\), where \(n\) is the number of activist funds, and \(m\) the number of quant funds. The left panel plots relative to the number of activists \(n\), while the right panel plots relative to the number ofquants \(m\). Parameter values are per Table [I].
Figure 6: Fund Market Competition and Fees: Base Parameters. The figure illustrates the relationship between endogenous managed fund fees and the state of the managed fund market for baseline parameter values, per Table [I]. Activist and quant market composition, in the bottom right panel, is illustrated using each fund type’s share of total fund AUM.
Figure 7: Fund Market Competition and Index Fees. The figure illustrates the relationship between mean managed fund fees and the state of the managed fund market for increasing passive index fee, $\bar{\pi}$, as plotted in the bottom left panel. Remaining parameter values are per Table 1. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.
Figure 8: Vary Index Fee. The effect of increasing the passive index fee, $\bar{\pi}$, is illustrated in the above figure. In the left column, all other parameters are per the baseline in Table 1. In the right column, baseline parameters are modified to have fund manager reservation utility $\zeta = 0.08\%$, exogenous quant exit rate $\theta_Q = 1$, and exogenous activist exit rate $\theta_A = 0.2$. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.
Figure 9: Vary Search Costs The figure shows the effects of varying search costs. The left panel varies activist search costs ($\zeta_A$), while the right panel varies quant search costs ($\zeta_Q$). Remaining parameter values are per Table 1. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.
Figure 10: Vary Depreciation (Efficiency). The figure shows the effects of varying the capital depreciation rate, capturing how efficiently different firms are run. The left panel varies depreciation of firms run by activists ($\delta_A$), while the right panel varies depreciation of firms run by quants ($\delta_Q$). Remaining parameter values are per Table 1. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.