

# Volatility Risk Pass-Through\*

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## Abstract

We develop a novel measure of volatility pass-through to assess international propagation of output volatility shocks to macroeconomic aggregates, equity prices, and currencies. An increase in country's output volatility is associated with a decrease in its output, consumption, and net exports. The average consumption pass-through is 50% (a 1% increase in output volatility increases consumption volatility by 0.5%) and it increases to 70% for shocks originating in smaller countries. The equity volatility pass-through is 90%, whereas the link between volatility of currency and fundamentals is weak. A novel channel of risk sharing of volatility risks can explain our empirical findings.

*Keywords:* Volatility pass-through, foreign exchange disconnect, risk sharing.

*JEL classification:* C62; F31; G12.

*First Draft:* February 1, 2015. *This draft:* November 12, 2018.

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\*We are grateful for early feedback to Nick Bloom, Tarek Hassan, Brent Neiman, and Matteo Maggiori. We thank our discussants: Alan De Genaro, Pasquale Della Corte, Francois Gourio, Olivier Jeanne, Xiaoji Lin, Galip Kemal Özhan, Alessandro Rebucci, Andreas Stathopoulos, Andrea Vedolin, and Irina Zviadadze. We also thank the participants at the AEA meeting, the SED meeting, the Wharton International Finance seminar, the International Macro-Finance Conference (Booth), the SITE summer session on uncertainty, the Second Workshop on Uncertainty hosted by UCL, the international workshop at the Hanqing Advanced Institute of Economics and Finance (Renmin University), the UBC Winter Finance Conference, the WFA Meetings, the SOFIE meetings, the SAFE Asset Pricing Workshop in Frankfurt, the Annual Meeting of the SEA, the Winter Meeting of the Econometric Society, the Summer Meeting of the Econometric Society, the Chicago Federal Reserve, the Università della Svizzera Italiana, BI, JHU, Ohio State University, the Annual International Finance Conference in Oslo, the Brazilian Meeting of Finance, and the CEBRA Conference in Warsaw and Madrid. We are grateful to The Rodney L. White Center for Financial Research for financial support.

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# 1 Introduction

The end of the Great Moderation period has highlighted once more the relevance of uncertainty shocks as key determinants of both economic activity and wealth in whole global economy. In this paper, we estimate and explain the international transmission of output volatility shocks to both currencies and international quantity dynamics. More precisely, focusing on a large cross section of major industrialized countries, we identify news to the conditional volatility of output, consumption, and real exchange rates. From this investigation we document several novel empirical findings. First, consumption and output volatilities are imperfectly correlated within countries. This implies that the growth rate of consumption in each country can experience changes in its conditional volatility that go beyond the arrival of endowment volatility shocks. Second, consumption volatility is more cross-country correlated than output volatility, suggesting that the output volatility shocks of one country propagate to the consumption of other countries.

To formalize the international propagation of output volatility shocks, we construct an index of volatility pass-through between two countries. Our index is equal to zero if a local output volatility shock results exclusively in an increase of local consumption volatility, without spilling over to the other country. Conversely, our index takes the value of one if a local output volatility shock results in an equal adjustment of consumption volatility in both countries. Our novel index measure allows us to address the following aspects of international risk sharing: (i) quantify the pass-through of macroeconomic uncertainty shocks across countries; (ii) examine the role of the country size (Hassan (2013)); (iii) relate volatility spillovers to external imbalances (Della Corte, Riddiough, and Sarno (2016)); and (iv) show the evidence for macroeconomic uncertainty risk sharing in international asset prices and currencies (Lustig, Roussanov, and Verdelhan (2011, 2014), Della Corte, Sarno, and Tsiakas (2009, 2011), Mueller, Stathopoulos, and Vedolin (2017)).

We find that the pass-through of output volatility is sizeable, especially when the uncertainty shocks originate from the smallest countries in our cross section. Specifically, when we focus on G7 countries, the pass-through is on the order of 50%, regardless of the country in which the output volatility shock materializes. This figure is as large as that of the pass-through of output level shocks. When we also include the next 10 countries according to their share of world GDP (henceforth G17), we find that the pass-through from bigger countries to smaller countries declines, whereas the pass-through of a volatility shock originating from small countries to large ones becomes as great as 70%. That is, smaller countries can better share volatility shocks compared to larger

countries, by redistributing a bigger fraction of their uncertainty shocks to their trading partners.

We also study the connection between output volatility and asset prices by focusing on the propagation of output volatility shocks to both equity returns and exchange rate changes. We find that the financial pass-through, that is, the pass-through of output vol shocks to equity returns volatility is very significant and it has increased over time, similarly to the consumption volatility pass-through. We relate this trend to increasing financial integration and risk-sharing in our model.

Our last empirical finding refers to the disconnect between the volatility of consumption differentials and the volatility of exchange rates. We document that the correlation of these volatilities is about 20% for the set of countries that we consider in our empirical investigation. Equivalently, the time-variation of exchange rate volatility is only partially driven by time-variation in the volatility of consumption aggregates. This is a novel observation that goes beyond the low correlation of the levels of consumption differentials and exchange rates (the Kollmann (1991) and Backus and Smith (1993) puzzle).

In the second part of this manuscript, we show that our main findings are an anomaly in the context of an equilibrium risk-sharing model with time-additive preferences. In contrast, when agents have recursive preferences, news about both future growth rates and future uncertainty are priced, and thus they can jointly affect trade and volatility dynamics in a manner more consistent with the data.

Specifically, we consider an economy with two countries, each populated by one agent with Epstein and Zin (1991) preferences (henceforth EZ preferences). Each agent is endowed with the stochastic supply of one country-specific good, whose dynamics are characterized by the presence of time-varying volatility shocks. Preferences feature a bias for the consumption of the domestic good. Trade occurs in frictionless goods markets and in financial markets featuring a complete set of state- and date-contingent securities.

Preferences are calibrated so that our agents dislike volatility of their continuation utilities. Since continuation utilities are a reflection of the entire future streams of consumption, we say that agents dislike long-run consumption variance. When news shocks hit the economy, agents have an incentive to trade in order to reduce the uncertainty of their future utility. Specifically, a country affected by a positive news shock will receive a smaller share of resources and have lower volatility of continuation utility going forward, but it will also have higher short-run consumption volatility.

When news pertains to future expected growth rates, the international reallocation

of resources results in an international exchange of both short-run and long-run consumption volatility across countries. That is, variances are characterized by negative comovements. We call this force the reallocation effect. News to output volatility, in contrast, produces a positive comovement in consumption volatilities across all countries: changes in output volatility spread in the cross section of countries, with the reallocation channel only partially mitigating the effects of local shocks on local consumption volatility.

The recursive risk-sharing arrangement that we described above is the key driver of our main results. Since agents dislike time variation in the volatility of their consumption, they actively trade with each other in order to dampen the associated change in the volatility of consumption following an output volatility shock. This reallocation results in a marked degree of volatility pass-through, which brings our model closer to the data.

Because of the concavity of the utility function with respect to country size, the reallocation channel is more pronounced for small countries than for large countries. As a result, our model predicts that shocks to output volatility should come with a larger pass-through when they affect small countries, consistent with the data. In a model with CRRA preferences, however, this result is missing, as volatility shocks are not directly priced and the associated risk-sharing motive is absent.

Furthermore, the model can account for the small extent of positive comovement between the volatility of consumption differentials and the volatility of exchange rate fluctuations thanks to two opposite forces. Volatility shocks tend to create a positive correlation between the two volatilities, as they increase the uncertainty of all the variables in the economy. Long-run shocks, in contrast, generate a large negative comovement.

To better understand the role of long-run shocks, we note that they are responsible for most of the fluctuations of the wealth distribution, that is, our reallocation channel. As the wealth distribution becomes more unequal, our countries depend more on each other in order to share risks. In equilibrium, they engage in more active trading, and their stochastic discount factors become more correlated. By no arbitrage, the real exchange rate becomes less volatile. Simultaneously, the reallocation effect makes the cross-country difference of the consumption growth rates more volatile, as the pass-through of consumption volatility is not symmetric across countries with different wealth shares.

In a model without shocks to output volatility (e.g., Colacito and Croce (2013)), the volatility of the exchange rate and that of the international differential of consumption growth rates would be strongly negative because of the dominance of the reallocation channel. In contrast, exogenous output volatility shocks increase the conditional volatil-

ity of all macroeconomic aggregates and hence endogenously produce positive comovements. Under our benchmark calibration, these opposite forces end up producing a positive but moderate correlation between consumption differentials and exchange rate volatility.

The resulting correlation is not as low as in the data and we speculate that introducing frictions may be a valuable venue for future progress. Although our attention is focused on a frictionless risk-sharing setting with symmetric countries, we regard the introduction of frictions, heterogeneity, and market incompleteness into our model as an important direction for future research in this area (see, e.g., Maggiori (2017); Gabaix and Maggiori (2015); Ready, Roussanov, and Ward (2017); Lustig, Roussanov, and Verdelhan (2011; 2014); Sandulescu, Trojani, and Vedolin (2017); Lustig and Verdelhan (2018); Bakshi, Cerrato, and Crosby (2017)). These frictions may be important in addressing the empirical link with international capital flows (Gourinchas and Rey (2007), Gourio, Siemer, and Verdelhan (2014)).

**Related literature.** Our study is related to the growing body of literature that has investigated the macroeconomic foundations of international financial markets' fluctuations (see, inter alia, Lustig and Verdelhan (2007), Farhi and Gabaix (2016), Verdelhan (2010), Stathopoulos (2017), Mueller, Stathopoulos, and Vedolin (2017), Della Corte *et al.* (2016), Heyerdahl-Larsen (2015), Pavlova and Rigobon (2007; 2010; 2013)). In particular, we contribute to the understanding of how country size matters for the propagation of international shocks, as recently emphasized by Hassan (2013), and Hassan, Mertens, and Zhang (2015; 2016). We differ from these papers by explicitly introducing time-varying uncertainty in macroeconomic fundamentals and studying its effects on the optimal international risk-sharing arrangement.

Additionally, several papers have documented the relevance of higher-order moments in sharpening our understanding of currency dynamics. Gavazzoni, Sambalaibat, and Telmer (2013) argue that non-Gaussian dynamics of the stochastic discount factors are needed to reconcile the riskiness of currencies with the level of the interest rates. Berg and Mark (2018) show that the cross-country high-minus-low conditional skewness of the unemployment gap is a measure of global macroeconomic uncertainty and it constitutes a factor that is robustly priced in currency excess returns. Zviadadze (2017) augments a VAR of US consumption growth, inflation, and three-month nominal yield with a common stochastic volatility component and analyzes the relationship between the term structure of currency carry trade and US macroeconomic risk. Relative to her analysis, we are interested in how relative GDP volatility shocks propagate into relative

consumption volatility in the cross section of G-17 countries and propose a model that accounts for the way that volatility risk is internationally shared. Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2015), Lettau, Maggiori, and Weber (2014), and Chernov, Graveline, and Zviadadze (2018) study the role of downside risk for currency risk premia. We regard the introduction of rare events as an important generalization of this framework.

More broadly, our analysis relates to the recent literature examining the role of uncertainty both in the data and in economic models (see, among others, Bloom (2009); Della Corte, Sarno, and Tsiakas (2011); Justiniano and Primiceri (2008); Jurado, Ludvigson, and Ng (2015); Kollmann (2016); Jones, Manuelli, Siu, and Stacchetti (2005); and Gilchrist, Sim, and Zakrajsek (2014)). In an early contribution, Ramey and Ramey (1995) show that countries with higher volatility of GDP have lower growth in the future. Consistent with their cross-sectional evidence, we find that higher domestic output volatility is associated with a decline in relative consumption in the future. We differ from these studies for our attention to the propagation of vol shocks across countries, as opposed to the within-country propagation of vol shocks to the level of macroeconomic aggregates.

The international long-run risk literature has already documented the ability of long-lasting consumption news shocks to account for several empirical regularities of international asset prices (see, among others, Colacito (2008); Colacito and Croce (2013); and Bansal and Shaliastovich (2013)). We differ from this literature in at least two dimensions. First, we provide novel evidence on the diffusion of fundamental output volatility shocks to consumption and currencies. Second, we provide an equilibrium explanation of our findings through the lens of a frictionless risk-sharing scheme in which volatility shocks are priced.

**Organization of the paper.** In the next section we describe our empirical strategy and our novel findings concerning the cross section of volatilities of major industrialized countries. Sections 3 and 4 describe our model and its implications. Section 5 concludes the paper. The appendix contains additional robustness checks and the model's extensions.

## 2 Empirical Evidence

In this section we lay out our econometric approach to analyze the implications of relative movements in macroeconomic volatilities within and across major industrialized coun-

tries. Focusing on the volatility of output shocks, we provide novel empirical evidence on the extent to which these volatility risks are transmitted to the relative volatility of consumption. We refer to this concept as the *volatility pass-through*. Further, we provide evidence linking volatility movements to output levels, trade dynamics, and volatilities of equity returns and exchange rates. Our novel evidence has important implications for understanding the risk sharing across countries, and it represents a challenge for many existing international finance models.

## 2.1 Data Description

**Sources and sample.** Our empirical analysis is based on the cross section of the following 17 major industrialized countries, ranked by GDP size: the United States, Canada, France, Germany, Italy, Japan, the United Kingdom, Australia, Belgium, Denmark, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, and Switzerland. In this study, we refer to the group of the first seven countries as G7 and to the expanded set of countries as G17. We collect the national accounts, population, and CPI data for these countries from the Organization for Economic Cooperation and Development (henceforth OECD) database. The exchange rates, quoted as the US dollar price of the foreign currency, are from the Federal Reserve Economic Database (henceforth FRED) database. The macroeconomic data are seasonally adjusted, real, and per capita.

To be consistent with the endowment economy that we analyze in sections 3 and 4, we abstract away from both investment and public expenditure and compute aggregate output as the sum of consumption and net exports. Since our model is based on a frictionless risk-sharing scheme, we follow the common practice of letting our quarterly dataset range from 1971:q1 to 2013:q4, a period of substantial financial integration across all major industrialized countries (see, among others, Quinn (1997), Obstfeld (1998), Taylor (2002), and Quinn and Voth (2008)).<sup>1</sup>

**Cross-sectional similarities and differences.** In table 1 we show key moments of our international data. For ease of exposition, we report cross-sectionally aggregated moments, as opposed to country-level values, within the G7 and G17 country groups. For G7 countries, we report simple averages of our aggregates. For G17 countries, we present both simple and GDP-weighted cross-sectional averages of our moments. To

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<sup>1</sup>Due to data availability and quality issues, the data for Belgium, Norway, and Spain start in 1981; for New Zealand in 1986; and for Portugal in 1991. Our Bayesian methods can easily be applied to an unbalanced panel.

**Table 1: Data Summary Statistics**

	G7 Avg.	G17 Avg.		G17 Quintile	
	Simple	Simple	Weighted	1 <sup>st</sup>	4 <sup>th</sup>
<i>Consumption growth</i>					
Mean	1.91	1.63	1.89	1.26	2.02
Std. Dev.	1.75	1.99	1.67	1.34	2.47
AR(1)	0.11	0.07	0.17	-0.16	0.31
<i>Output growth</i>					
Mean	1.94	1.71	1.93	1.43	2.00
Std. Dev.	2.21	2.97	2.02	2.01	4.43
AR(1)	0.00	-0.09	0.07	-0.26	0.09
<i><math>\Delta</math>Net Exports over Output:</i>					
Mean	0.03	0.08	0.04	-0.30	0.34
Std. Dev.	1.60	2.48	1.45	1.79	3.24
AR(1)	0.00	-0.09	0.07	-0.26	0.09
<i>Within-Country Correlations:</i>					
Consump. and output growth	0.67	0.51	0.71	0.35	0.72
Consump. and output vol	0.54	0.47	0.65	0.26	0.80
<i>Across-Country Correlations:</i>					
Consump. growth	0.27	0.24	0.25	0.13	0.33
Output growth	0.15	0.14	0.14	0.06	0.20
Consump. vol	0.51	0.47	0.45	0.35	0.66
Output vol	0.32	0.30	0.30	0.18	0.45

Notes: This table shows summary statistics for consumption growth, output growth, change in net-export-to-output ratio, and the correlations of consumption and output volatilities within and across countries. ‘G7 Avg.’ (‘G17 Avg.’) refers to simple (both simple and GDP-weighted) averages of key moments for G7 (G17) countries. The rightmost two columns show the first and fourth quintiles of the moments of interest in the G17 cross section. Macroeconomic variables are seasonally adjusted, real, and per capita. Means and volatilities are annualized, in percentages. Quarterly observations are from the 1971:Q1–2013:Q4 sample.

assess the extent of cross-country heterogeneity, for each moment we also report the 1<sup>st</sup> and 4<sup>th</sup> quintiles within the G17 group.

We highlight three relevant facts. First, the moments for the G7 group are very similar to the typical estimates for the United States. As an example, consumption growth has a mean of about 2% per year and a volatility of about 1.75%. In the G17 aggregate, the average growth rate declines, whereas the unconditional volatility of both output and consumption increases. In both cases, however, changes are relatively mod-



est. Both quarterly consumption and output growth are almost serially uncorrelated.

Second, the average change in the net-export-to-output ratio is distributed nearly symmetrically around zero. In the group of G17 countries, this moment ranges from  $-30\%$  to  $+34\%$ . Since smaller countries have more volatile output than bigger countries, they also tend to have more volatile net-export-to-output ratios.

Third, in both the G7 and G17 groups, consumption growth rates feature low international correlations.<sup>2</sup> Further, output and consumption growth rates are imperfectly correlated within countries. These empirical findings on risk sharing, alongside with other macroeconomic and financial market moments, are going to be important targets to assess our economic model.

In the next sections, we describe in detail our identification of the time-varying volatility components and address their comovements within and across countries.

## 2.2 Volatility Measurement and Comovements

We extract the volatility of the series of interest,  $z_t$ , by estimating the following specification:

$$\begin{aligned} z_t &= \mu(1 - \rho) + \rho z_{t-1} + e^{\sigma_t(z)/2} \eta_t, \\ \sigma_t(z) &= \mu_\sigma(1 - \nu) + \nu \sigma_{t-1}(z) + \sigma_w w_t, \end{aligned} \tag{2.1}$$

where  $\sigma_t(z)$  is a latent process equal to the logarithm of the variance of macroeconomic shock to  $z_t$ . The innovations  $\eta_t$  and  $w_t$  are independent Gaussian shocks to the level and the volatility of  $z_t$ , respectively. The parameters  $\rho$  and  $\nu$  govern the persistence of  $z_t$  and  $\sigma_t(z)$ , respectively, whereas  $\mu$  and  $\mu_\sigma$  represent the average level and volatility of  $z_t$  and  $\sigma_t(z)$ , respectively. The parameter  $\sigma_w$  captures the volatility of volatility. According to our specification, the variance of  $z_t$  is guaranteed to take on positive values. In untabulated tests we directly estimated volatility in levels, with very similar results. For this reason, in the remainder of this manuscript we refer to  $\sigma_t$  as either log-volatility or volatility interchangeably.

Similar volatility specifications are employed in Cogley and Sargent (2005) and Primiceri (2005) in the context of macroeconomic volatility, and in Della Corte, Sarno, and Tsiakas (2009) for the financial volatility modeling. For robustness, we have also entertained more general specifications which allow for i) a correlation between the level and volatility shocks, and ii) additional explanatory variables in the level equation. The

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<sup>2</sup>The quantity anomaly in Backus, Kehoe, and Kydland (1994) does not apply to our dataset, as our measured output excludes both investment and government expenditure.

correlations of the filtered volatility series across these specifications with the benchmark exceed 90% for all the countries, and all the key implications for the pass-through are unchanged, as shown in the Appendix A.

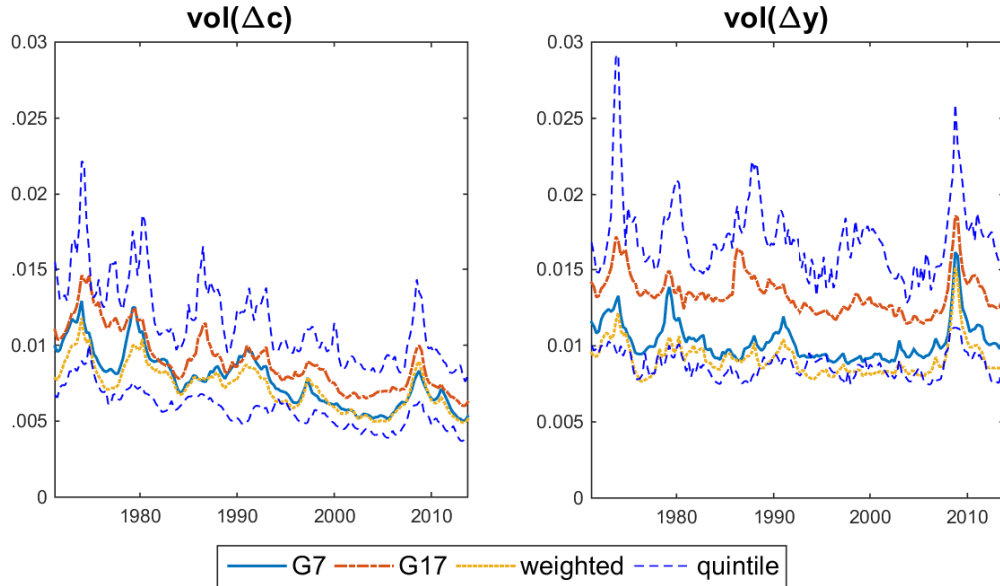
We estimate the system of equations (2.1) following the Bayesian methods in Kim, Shephard, and Chib (1998). For each country, we fit our volatility specification to aggregate consumption and output growth separately. To check the robustness of our results, we also employ a specification in which the volatility parameters are restricted to be common across countries and are jointly estimated in our cross section of countries. For parsimony, a complete summary of the estimation details is provided in the appendix.

**Volatilities: aggregate time pattern.** In figure 1, we show our fitted volatilities aggregated across both G7 and G17 countries. For the G17 group, we also plot the first and the fourth cross-sectional volatility quintiles. Consistent with the findings reported in table 1, consumption volatility is systematically lower than output volatility. Further, our estimation procedure captures the well-documented Great Moderation phenomenon, as both our estimated consumption and output volatilities slowly decline from the 1980s to the mid-2000s. These findings are consistent with those documented by Lettau, Ludvigson, and Wachter (2008), Stock and Watson (2002) and McConnell and Quiros (2000) for the United States.

Consistent with the unconditional evidence in table 1, G17 countries have a larger average volatility level relative to the G7 group. In both country groups, our conditional estimates exhibit substantial and persistent fluctuations over time. More broadly, the time pattern of the estimated aggregate volatilities shares similar characteristics across G7 and G17 countries. These results suggest that our novel findings on international volatility comovements are quite general, as they apply to a large international cross section. We note that the correlation between our measure of output volatility for the US and the VIX is 0.5. More broadly, we find such a positive but imperfect degree of correlation using realized volatilities in our entire cross section of countries. Thus our assessment of macroeconomic volatility is related to and yet distinct from financial volatility.

**Volatilities: comovements.** Uncertainty shocks appear to be modestly correlated across countries for both consumption and output, and the correlation structure of the volatilities mimics that of the levels.

Specifically, table 1 shows that the cross-country correlation of endowment volatilities is about 0.30, a number close to the cross-country correlation of the levels of the growth



**Figure 1 - Macroeconomic Volatilities.** This figure shows estimates of macroeconomic volatilities of real consumption and output growth. Volatilities,  $e^{\sigma_t/2}$ , are estimated at a country level according to equation (2.1). The G7 line shows the equally weighted cross-sectional average for G7 countries. “G17” reports the equally weighted average across all the G17 countries. “Weighted” reports the GDP-weighted average across G17 countries. Dashed lines show the first and fourth quantiles of the volatilities in the G17 cross section. Quarterly observations range from 1971:Q1 to 2013:Q4.

rates. The cross-country correlation of consumption volatilities is slightly higher than that of output volatilities, once again consistent with that observed for the growth rates of the levels. Within each country, in contrast, the volatilities of consumption and output comove strongly with each other. Their correlation is 0.70, a figure similar to that of the consumption and output growth rates.

In our next step, we adopt a VAR approach to (i) better characterize the joint dynamics of both levels and volatilities, and (ii) quantify the pass-through of volatility shocks.

### 2.3 Volatility Risk Pass-Through

**Relative volatility shocks.** To evaluate the dynamic impact of shocks to relative volatility ( $\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US})$ ) across countries, we jointly estimate the following  $N$  countries VAR(1):

$$\tilde{Y}_{t,i} = \tilde{\mu}_{Y,i} + \tilde{\Phi} \tilde{Y}_{t,i} + \tilde{\Sigma} \tilde{u}_{t,i}, \quad i = 1, 2, \dots, N \quad (2.2)$$

where

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix}, \quad (2.3)$$

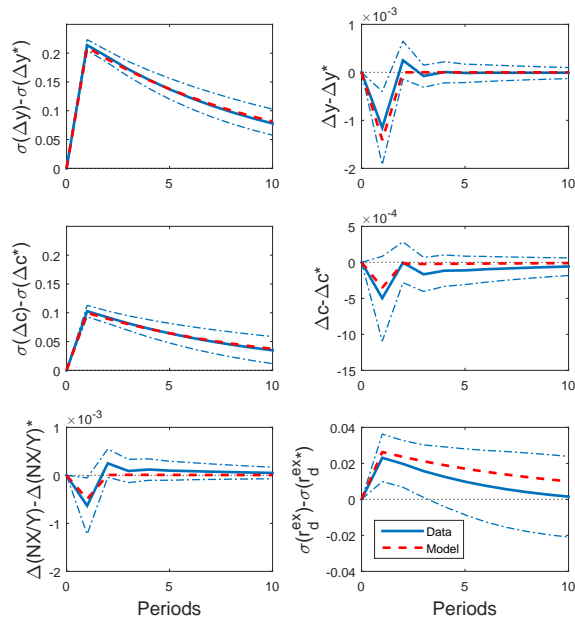
where  $\Delta y_i - \Delta y_{US}$ ,  $\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US})$ ,  $\Delta c_i - \Delta c_{US}$ , and  $\Delta(NX/Y)_i - \Delta(NX/Y)_{US}$  denote the difference between country  $i$  and the US in growth rates of endowments; the volatilities of consumption growth rates; the growth rates of consumption; and the net-export-to-output ratios, respectively. We note that  $N$  is equal to 6 for G-7 countries and 16 for G-17 countries.

In Appendix A.1, we show that our key results are robust both to different specifications and estimation procedures, and to the choice of a global benchmark, rather than considering just the US. Furthermore, our results are virtually unchanged when we account for heterogenous exposure to a common global volatility process across countries.

Since we adopt the US as the baseline home country throughout our main text analysis, this specification allows us to focus on relative bilateral adjustments computed with respect to a common benchmark. To sharpen the system's identification, we assume that the fundamental persistence and volatility parameters  $\tilde{\Phi}$  and  $\tilde{\Sigma}$  are common across countries, whereas the intercepts  $\tilde{\mu}_{Y,i}$  are allowed to be country specific. Under these assumptions, we can estimate the VAR parameters by pooling the demeaned data across countries. We estimate the system of VAR equations as Seemingly Unrelated Regressions.

Using our estimated VAR, we can trace the relative response of the macroeconomic variables to an increase in output volatility in the foreign country relative to the US. In the main text, we report the results under a lower diagonal Cholesky decomposition of the variance-covariance matrix for the setting in which output volatility shocks are ranked before the other macroeconomic aggregates of interest. Our main results also hold when volatility is ranked after the output growth (see Appendix A, table A1). In this sense, we do not take a stand on whether volatility shocks causes level shocks or viceversa, but rather we assess the role of volatility shocks orthogonal to level shocks.

In figure 2, we show the estimated impulse responses for the G7 countries to a relative volatility shock. In table 2, we report the contemporaneous responses of all the variables in the system to this type of shock. These numbers correspond to the entries in the first column of the matrix  $\tilde{\Sigma}$  in equation (2.2). We perform this analysis for both the G7 and



**Figure 2 - Responses to a Relative Volatility Shock.** This figure shows the estimates of the relative responses of the volatility and growth rate of output ( $\Delta y$ ), the volatility and growth rate of consumption ( $\Delta c$ ), and the change of net-export-to-output ratio ( $\Delta NX/Y$ ), and the volatility of excess returns ( $r_d^{ex}$ ) to a one-standard-deviation increase in the volatility of output in the foreign country relative to the US. Dashed (dotted) lines refer to the point estimates (95% credible interval) of the VAR(1) specified in equation (2.3) with the addition of equity returns volatility differential,  $\sigma_t(r_{US,t}^{ex}) - \sigma_t(r_{i,t}^{ex})$ . Solid lines show the output from our model under the benchmark quarterly calibration reported in table 4.

the remaining G17 countries (hereafter, the bottom-10 G17). Our empirical evidence highlights several important cross-sectional aspects of volatility shocks across countries.

First, when country  $i$  experiences an increase in its output volatility relative to the US, both its relative consumption and output growth rates fall. The estimated effects are large and almost always statistically significant. For example, in our G7 specification, foreign output growth falls by nearly half a percentage point relative to the US upon the realization of a one-standard-deviation relative volatility shock. These findings complement the one-country evidence in Bansal, Kiku, Shaliastovich, and Yaron (2014) and Bloom (2009) in showing that an increase in domestic volatility decreases real economic activity. For the same country group, the fall in the relative level of consumption growth is about 0.20%, that is, half of that of output. This mitigation happens through net imports, as the country with the highest volatility shock experiences a deterioration of its current account.

Second, following a relative increase in output volatility, the volatility of consumption increases as well. We find it convenient to explore this effect in greater detail by defining

**Table 2: Volatility Risk Pass-Through**

<b>Panel A: Contemporaneous adjustments to relative volatility shocks</b>					
$\sigma(\Delta y)$	$\Delta y$	$\sigma(\Delta c)$	$\Delta c$	$\Delta(NX/Y)$	Pass-through
<i>US/G7 Countries:</i>					
0.21	-0.46	0.10	-0.20	-0.25	0.52
[0.20; 0.22]	[-0.77; -0.17]	[0.09; 0.11]	[-0.44; 0.03]	[-0.49; -0.02]	[0.48; 0.56]
<i>US/Bottom-10 G17 Countries:</i>					
0.21	-0.57	0.08	-0.16	-0.39	0.61
[0.21; 0.22]	[-0.95; -0.19]	[0.07; 0.09]	[-0.41; 0.09]	[-0.73; -0.06]	[0.56; 0.65]
<b>Panel B: Pass-through and size</b>					
			Origin of Vol. Shock:		
			U.S.	Foreign Country	
<i>US/G7 Countries:</i>			0.49	0.57	
			[0.43; 0.54]	[0.51; 0.63]	
<i>US/Bottom-10 G17 Countries:</i>			0.51	0.72	
			[0.45; 0.57]	[0.66; 0.78]	

Notes: Panel A shows the estimates of the contemporaneous responses ( $\tilde{\Sigma}_{1j}$ ) of the VAR(1) specified in equations (2.2)–(2.3) with respect to a shock to relative output volatility. Responses of output growth, consumption growth, and net-exports-to-output ratio are annualized, in percentages. Volatility pass-through is defined as in equation (2.4). Panel B reports pass-through measures based on the estimates of the VAR in equations (2.6)–(2.7) with respect to volatility shocks affecting either the US or the remaining countries. We report 95% credible intervals in brackets. Our quarterly data range from 1971:q1 to 2013:q4.

a volatility pass-through index as follows:

$$\text{Pass-through} := 1 - \frac{\partial(\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}))}{\partial(\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}))}. \quad (2.4)$$

Since our analysis is based on country pairs, this index is equal to zero if an increase in (log) output volatility in one country results in a one-for-one increase in its own (log) consumption volatility.<sup>3</sup> If instead an output volatility shock results in an equally redistributed increase in consumption volatility across the two countries, the volatility pass-through is one.

In an economy with time-additive preferences defined over one good, perfect risk-sharing implies a pass-through of one, as consumption is equalized across all possible

<sup>3</sup>Since the volatilities in our VAR specification are in log-units, we note that the pass-through index is equivalent to 1 minus the elasticity of consumption volatility to GDP volatility. This means that, for example, a pass-through index of 0.75 can be interpreted as the ratio of consumption volatilities increasing by 0.25% in response to a 1% increase in the ratio of GDP volatilities.

states and hence  $\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) = 0 \forall t$ . Vice versa, in an endowment economy in which countries are subject to autarky, that is, they cannot trade, our index is equal to zero, as  $C_{i,t} = Y_{i,t} \forall i, t$  and hence  $(\sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US})) = (\sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US})) \forall i, t$ .

We can compute the pass-through index directly from the estimate of the VAR in equations (2.2)–(2.3), as

$$\text{Pass-through} = 1 - \tilde{\Sigma}_{3,1} / \tilde{\Sigma}_{1,1}. \quad (2.5)$$

Our estimates suggest that the volatility pass-through is about 50% for G7 countries, meaning that if country  $i$  receives a country-specific output volatility shock of 1%, its own consumption volatility goes up by just 0.5%. This index increases further to 60% when we focus on smaller countries, suggesting that the international sharing of volatility shocks is more relevant for this set of countries.<sup>4</sup>

**Country-specific shocks.** The specification of the VAR in equation (2.3) is parsimonious, and provides direct evidence on the magnitude of the pass-through of the volatility shocks in the data. In this section, we extend our analysis to (i) provide information on the size and correlation of shocks across countries, and (ii) document size effects for the pass-through; that is, difference in responses to volatility shocks arising from big versus small countries. The former is relevant for model calibration purposes, while the latter extends our understanding of volatility shock risk-sharing in the data.

Specifically, we propose an extended VAR,

$$Y_{t,i} = \mu_{Y,i} + \Phi Y_{t-1,i} + \Sigma u_{t,i}, \quad (2.6)$$

in which we disentangle foreign and U.S. variables:

$$Y'_{i,t} = \left[ \sigma_t(\Delta y_i) \quad \sigma_t(\Delta y_{US}) \quad \Delta y_i \quad \Delta y_{US} \quad \sigma_t(\Delta c_i) \quad \sigma_t(\Delta c_{US}) \right]. \quad (2.7)$$

As before, the persistence and scale matrices are common across countries, whereas the intercepts pick out country-specific differences in the means. For parsimony, we consider the smallest set of variables required for both calibration reasons and for the assessment of the volatility pass-through. As a result, we exclude both the change in net exports

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<sup>4</sup>In untabulated results, we have checked the robustness of our findings to alternative definitions of GDP. Our evidence on the pass-through index is robust to the inclusion of investments (it is about 50%), and it becomes slightly larger if we also include public expenditure (about 78%). Since the model that we propose abstracts away from investments and public expenditures, we use the numbers reported in table 2 as our benchmark.

and the consumption growth rates from this VAR.

The estimation results used to guide our calibration are discussed in the next section and are reported in table 4. In panel B of table 2, we report the implied volatility pass-through due to either a one-standard-deviation increase in US output volatility or a one-standard-deviation in foreign output volatility for both the G7 and the bottom-10 G17 countries. Specifically, according to our VAR specification in equations (2.6)–(2.7), the pass-through for US-originated shocks is

$$\text{Pass-through}^{US} = 1 - (\tilde{\Sigma}_{6,2} - \tilde{\Sigma}_{5,2})/\tilde{\Sigma}_{2,2}. \quad (2.8)$$

For non-US shocks, we adopt the same expression, but we change the order of variables in our VAR from what reported in (2.7) to

$$Y'_{i,t} = \left[ \sigma_t(\Delta y_{US}) \quad \sigma_t(\Delta y_i) \quad \Delta y_{US} \quad \Delta y_i \quad \sigma_t(\Delta c_{US}) \quad \sigma_t(\Delta c_i) \right],$$

and retain the definition of pass-through provided in (2.8).

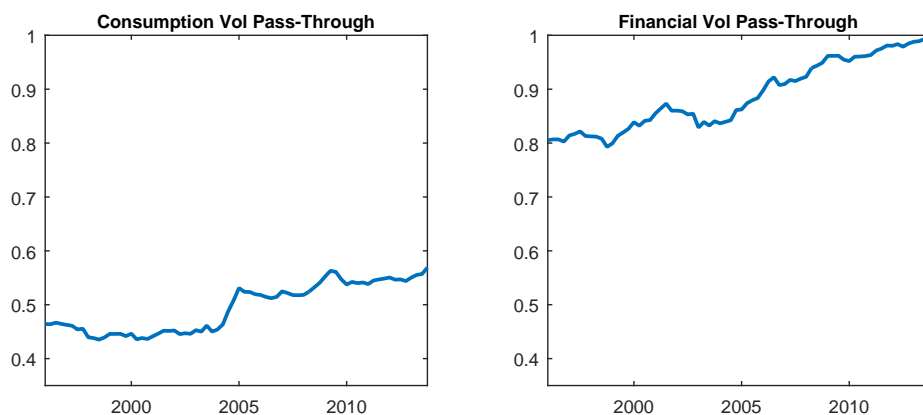
The additional insight provided by this estimation is that the pass-through is sensitive to the size distribution of the countries that we analyze. When we focus on the G7 group, all countries tend to have a similar size and a pass-through in the common range (51%–54%), regardless of the origin of the volatility shock. In contrast, when we focus on the US versus the bottom-10 G17 countries, i.e., a cross section with more dispersion in size, the origin of the shock matters. We find that the volatility pass-through is larger if the volatility shock originates from the smaller economies.

According to our estimates, the bottom-10 G17 countries have a pass-through of 72% when they receive an adverse output volatility shock. When the US receives a volatility shock, in contrast, the pass-through to these smaller countries is just 51%, a number comparable to that estimated for the other G7 countries. All together, these results suggest a novel empirical finding: after a spike in endowment uncertainty, small countries mitigate their consumption volatility better than large countries.

To highlight further the scope of our findings, we compare our volatility pass-through measure to a measure of level pass-through defined as  $1 - \partial(\Delta c_i - \Delta c_{US}) / \partial(\Delta y_i - \Delta y_{US})$ . Untabulated results show that our volatility pass-through measures are as sizeable as their level pass-through analogues.

**Financial pass-through.** We enrich further our analysis by studying the pass-through of endowment volatility shocks to the volatility of equity excess returns,  $r_{i,t}^{ex}$ . Specifically, we estimate  $\sigma_t(r_{i,t+1}^{ex})$  according to equation (2.1) and add it as the last variable to





**Figure 3** - Time-Varying Pass-Through. This figure shows time-varying measures of pass-through for both consumption and equity excess returns. The consumption volatility pass-through index is computed as specified in equation (2.4) on a rolling-window sample that comprises 100 observations. The financial pass-through refers to the impact of output vol shocks on equity return volatility. Quarterly observations are from the 1971:Q1–2013:Q4 sample. All data refer to G7 countries.

our VAR vector of states in equation (2.3). The financial pass-through is then defined similarly to the consumption pass-through by replacing  $\tilde{\Sigma}_{3,1}$  with  $\tilde{\Sigma}_{6,1}$  in equation (2.5).

On average the financial pass-through is quite large: our estimated value for the financial pass-through is about 0.9 (with a confidence interval of [0.83;0.95]), that is, almost twice as large as our estimated consumption pass-through index.<sup>5</sup>

Further, we find that the amount of pass-through varies over time, and it is increasing over our sample. To measure time-variation in volatility pass-through, we re-estimate our extended VAR in a rolling-window fashion, and compute conditional estimates of the pass-through index. In figure 3, we show that the pass-through of both consumption and equity return volatility has been increasing over time. This is an interesting empirical observation that, through the lens of our model, can be attributed to increasing financial integration and risk sharing.

We further analyze whether the time-variation in the pass-through is related to macroeconomic factors, such as global recessions. We construct a recession index that counts the number of G7 countries in recession in a given quarter.<sup>6</sup> We then regress our detrended pass-through measures on the average number of countries in recession over the same rolling window used to compute the pass-through indices. For robustness, we

<sup>5</sup>The unconditional estimate of the financial pass-through is consistent with the impulse response evidence in the bottom-right panel of figure 2. Results are nearly identical whether we focus on US/G7 countries or US/Bottom-10 G17 Countries.

<sup>6</sup>We classify a country as being in a recession if it has experienced negative real output growth for at least two consecutive quarters.

**Table 3: Volatility Disconnect**

	G7 Avg.	G17 Avg.		G17 Quintile	
	Simple	Simple	Weighted	1 <sup>st</sup>	4 <sup>th</sup>
<i>Levels Disconnect</i>					
$corr(\Delta cd_{t+1}, \Delta e_{t+1})$	-0.14	-0.11	-0.13	-0.19	-0.04
$corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$	-0.14	-0.17	-0.14	-0.29	-0.05
<i>Volatility Disconnect</i>					
$corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$	0.20	0.21	0.20	-0.01	0.42
$corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$	0.27	0.25	0.26	-0.02	0.52

Notes: This table shows correlations between the level and conditional volatility of consumption growth differentials ( $cd_t^i \equiv \Delta c_t^{US} - \Delta c_t^i$ ) and exchange rate growth ( $\Delta e_t^{i|USD}$ ), respectively. In both cases, the US is considered the benchmark home country. Cumulative growth rates are denoted by ‘ $\widehat{\cdot}$ ’. ‘G7 Avg.’ (‘G17 Avg.’) refers to simple (both simple and GDP-weighted) averages of key moments for G7 (G17) countries. The rightmost two columns show the first and fourth quintiles of the moments of interest in the G17 cross-section. Consumption is seasonally adjusted, real, and per capita. Volatility estimates are based on the specification reported in equation (2.1). Quarterly observations are from the 1971:Q1–2013:Q4 sample.

also replace our global recession index by the NBER recession dummy. In all cases, we find that the time-variation in the pass-through measures are unexplained by recessions and is stable over business cycle frequencies.

**The volatility disconnect.** Given our focus on the dynamics of volatility, we push our analysis one step further and study the connection between the conditional variance of consumption growth differentials and the conditional variance of exchange rate movements. To the extent to which consumption differentials capture risk-sharing opportunities, time-varying uncertainty in consumption differentials should be an important determinant of time-varying exchange rate uncertainty.

As shown in the bottom portion of table 3, empirically this correlation is very modest and ranges between 20 and 30%. In absolute value, these magnitudes are similar to those for the level shocks (top portion of the same table). We call this novel empirical fact the *volatility disconnect*, in the spirit of the literature (Backus and Smith (1993)).<sup>7</sup> We consider this empirical result as a further interesting restriction for international macroeconomic models.

To summarize, our evidence shows that output volatility shocks decrease relative output and consumption across countries and increase consumption volatility. In relative terms, the effects for the consumption growth rate are smaller than for output growth

<sup>7</sup>In untabulated tests, we estimated a similar level of disconnect also with the volatility of output differentials. These results are available upon request.

rates, and the consumption volatility response is larger if output volatility shocks originate in a larger country. Equivalently, the pass-through from large to small countries is smaller than the pass-through from small to large countries. Both the consumption and equity returns volatility pass-through indices have increased over the last 20 years. Furthermore, we find a strong disconnect between currency volatility and consumption differentials volatility. In the next section, we develop an economic model that helps us to understand to which extent risk-sharing can, or fails to, explain our volatility evidence.

### 3 Model

The economy consists of two countries, home ( $h$ ) and foreign ( $f$ ), and two goods,  $X$  and  $Y$ . Agents' preferences are defined over consumption aggregates of the two goods as follows.

**Consumption aggregate.** Let  $x_t^i$  and  $y_t^i$  denote the consumption of good  $X$  and good  $Y$  in country  $i \in \{h, f\}$  at date  $t$ . Let  $\alpha \in (0, 1)$ . The consumption aggregates in the home and foreign countries are

$$C_t^h = (x_t^h)^\alpha (y_t^h)^{1-\alpha} \quad \text{and} \quad C_t^f = (x_t^f)^{1-\alpha} (y_t^f)^\alpha, \quad (3.1)$$

respectively. The parameter  $\alpha$  captures the degree of bias of the consumption of each representative agent. In what follows we assume that the home country is endowed with good  $X$ , while the foreign country is endowed with good  $Y$ . Following some of the international macrofinance articles surveyed by Lewis (2011), we assume that  $\alpha$  is larger than 0.5. This allows us to build consumption home bias into the model.

**Preferences.** As in Epstein and Zin (1993), agents' preferences are recursive but not time separable:

$$U_t^i = \left[ (1 - \delta) \cdot (C_t^i)^{1-1/\psi} + \delta E_t \left[ (U_{t+1}^i)^{1-\gamma} \right]^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \quad \forall i \in \{h, f\}. \quad (3.2)$$

The coefficients  $\gamma$  and  $\psi$  measure the relative risk aversion (RRA) and the IES, respectively.

In contrast to the constant RRA case, these preferences allow agents to be risk averse in future utility as well as future consumption. The extent of such utility risk aversion

depends on the preference for early resolution of uncertainty, measured by  $\gamma - 1/\psi > 0$ . To better highlight this feature of the preferences, we focus on the ordinally equivalent transformation

$$V_t = \frac{U_t^{1-1/\psi}}{1 - 1/\psi}$$

and approximate it with respect to  $\theta \equiv \frac{\gamma-1/\psi}{1-1/\psi}$  around  $\theta_0 = 1$ :

$$\begin{aligned} V_t &= (1 - \delta) \frac{C_t^{1-1/\psi}}{1 - 1/\psi} + \delta E_t [V_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}} \\ &\approx (1 - \delta) \frac{C_t^{1-1/\psi}}{1 - 1/\psi} + \delta E_t [V_{t+1}] - \frac{\delta}{2} \frac{\theta}{E_t [V_{t+1}]} \text{Var}_t [V_{t+1}]. \end{aligned} \quad (3.3)$$

Note that the sign of  $\left(\frac{\theta}{E_t[V_{t+1}]}\right)$  depends on the sign of  $(\gamma - 1/\psi)$ . When  $\gamma = 1/\psi$ , the agent is utility-risk neutral and preferences collapse to the standard time-additive case. When the agent prefers early resolution of uncertainty, that is, when  $\gamma > 1/\psi$ , the coefficient  $\theta$  is positive: uncertainty about continuation utility reduces welfare and generates an incentive to trade off future expected utility,  $E_t [V_{t+1}]$ , for future utility risk,  $\text{Var}_t [V_{t+1}]$ .

This mean-variance trade-off is absent when agents have standard time-additive preferences, and it represents the most important element of our analysis, given our focus on the propagation of uncertainty shocks.

Since there is a one-to-one mapping between utility,  $U_t^i$ , and lifetime wealth, that is, the value of a perpetual claim to consumption,  $W_{c,t}^i$ ,

$$U_t^i = [(1 - \delta)(C_t^i + W_{c,t}^i)]^{\frac{1}{1-1/\psi}}, \quad \forall i \in \{h, f\}, \quad (3.4)$$

the optimal risk-sharing scheme can also be interpreted in terms of the mean-variance trade-off of wealth. For this reason, in what follows we use the terms “wealth” and “continuation utility” interchangeably.

**Endowments.** We choose to endow each country with a stochastic supply of its most-preferred good. Endowments are specified in the spirit of Colacito and Croce (2013), with the important difference of accounting also for time-varying risk:

$$\begin{aligned} \Delta \log X_t &= \mu_x + z_{1,t-1} + e^{\sigma_{x,t}/2} \sigma \varepsilon_{x,t} - c_{t-1} \\ \Delta \log Y_t &= \mu_y + z_{2,t-1} + e^{\sigma_{y,t}/2} \sigma \varepsilon_{y,t} + c_{t-1}, \end{aligned} \quad (3.5)$$

where the process  $ci_t \equiv \tau \log(X_t/Y_t)$  with  $\tau \in (0, 1)$  introduces cointegration and guarantees the existence of the equilibrium, and the components  $z_1$  and  $z_2$  are highly persistent AR(1) processes,

$$z_{j,t} = \rho z_{j,t-1} + \sigma_z \varepsilon_{j,t}, \forall j \in \{1, 2\}. \quad (3.6)$$

Throughout the paper, we refer to  $\varepsilon_{1,t}$  and  $\varepsilon_{2,t}$  as long-run shocks, due to their long-lasting impact on the growth rates of the two endowments. Similarly, we call  $\varepsilon_{x,t}$  and  $\varepsilon_{y,t}$  short-run shocks.

We focus on time-varying short-run risk, as captured by the following process:

$$\sigma_{j,t} = \rho_\sigma \sigma_{j,t-1} + \sigma_{sr} \varepsilon_{\sigma j,t}, \forall j \in \{x, y\}. \quad (3.7)$$

Shocks are jointly log-normal:

$$\xi_t \equiv \begin{bmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} & \varepsilon_{x,t} & \varepsilon_{y,t} & \varepsilon_{\sigma 1,t} & \varepsilon_{\sigma 2,t} \end{bmatrix} \sim i.i.d.N(\mathbf{0}, \Sigma),$$

and the matrix  $\Sigma$  is assumed to be block-diagonal to allow for cross-country correlation of shocks of the same type.

**Markets.** At each date, trade occurs in a complete set of one-period-ahead claims to state-contingent consumption. Financial and goods markets are assumed to be frictionless. The budget constraints of the two agents can be written as

$$\begin{aligned} x_t^h + p_t y_t^h + \int_{\zeta^{t+1}} A_{t+1}^h(\zeta^{t+1}) Q_{t+1}(\zeta^{t+1}) &= A_t^h + X_t \\ x_t^f + p_t y_t^f + \int_{\zeta^{t+1}} A_{t+1}^f(\zeta^{t+1}) Q_{t+1}(\zeta^{t+1}) &= A_t^f + p_t Y_t, \end{aligned} \quad (3.8)$$

where  $p_t$  denotes the relative price of goods  $X$  and  $Y$  (the terms of trade),  $A_t^i(\zeta^t)$  denotes country  $i$ 's claims to time  $t$  consumption of good  $X$ , and  $Q_{t+1}(\zeta^{t+1})$  gives the price of one unit of time  $t+1$  consumption of good  $X$  contingent on the realization of  $\zeta^{t+1}$  at time  $t+1$ . In equilibrium, the market for international state-contingent claims clears, implying that  $A_t^h + A_t^f = 0, \forall t$ . In our analysis, all assets are denominated in units of the numeraire good. We regard the extension to a setup in which the currency of denomination of international assets matters as an important direction for future research (see Maggiori, Neiman, and Schreger (2017) and Du, Pflueger, and Schreger (2017)).

**Prices.** The stochastic discount factor in consumption aggregate units is

$$M_{t+1}^i = \delta \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}^{i1-\gamma}}{E_t [U_{t+1}^{i1-\gamma}]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \quad (3.9)$$

Since markets are assumed to be complete, the log growth rate of the real exchange rate is

$$\Delta e_t = \log M_t^f - \log M_t^h \quad (3.10)$$

and the relative price of the two goods is  $p_t = \frac{(1-\alpha)x_t^h}{\alpha y_t^h}$ .

**Allocations.** Under complete markets, we can compute efficient allocations by solving the associated Pareto problem. The planner attaches date 0 nonnegative Pareto weights  $\mu^h = \mu$  and  $\mu^f = 1 - \mu$  to the consumers and chooses the sequence of allocations  $\left\{ x_t^h, x_t^f, y_t^h, y_t^f \right\}_{t=0}^{+\infty}$  to maximize

$$\Lambda = \mu \cdot U_0^h + (1 - \mu) \cdot U_0^f,$$

subject to the following sequence of economy-wide feasibility constraints:

$$\begin{aligned} x_t^h + x_t^f &= X_t \\ y_t^h + y_t^f &= Y_t, \quad \forall t \geq 0, \end{aligned}$$

where the state-dependent notation is omitted for the sake of clarity. In characterizing the equilibrium, we follow Anderson (2005) and formulate the problem using the ratio of time-varying pseudo-Pareto weights,  $S_t = \mu_t / (1 - \mu_t)$ , as an additional state variable. This technique enables us to take into account the nonseparability of the utility functions.

The first-order necessary conditions imply the following allocations:

$$\begin{aligned} x_t^h &= \alpha X_t \left[ 1 + \frac{(1-\alpha)(S_t-1)}{1-\alpha+\alpha S_t} \right], & x_t^f &= (1-\alpha) X_t \left[ 1 - \frac{\alpha(S_t-1)}{1-\alpha+\alpha S_t} \right] \\ y_t^h &= (1-\alpha) Y_t \left[ 1 + \frac{\alpha(S_t-1)}{\alpha+(1-\alpha)S_t} \right], & y_t^f &= \alpha Y_t \left[ 1 - \frac{(1-\alpha)(S_t-1)}{\alpha+(1-\alpha)S_t} \right], \end{aligned} \quad (3.11)$$

where

$$S_t = S_{t-1} \cdot \frac{M_t^h}{M_t^f} \cdot \left( \frac{C_t^h/C_{t-1}^h}{C_t^f/C_{t-1}^f} \right), \quad \forall t \geq 1 \quad (3.12)$$

and  $S_0 = 1$ , as we start the economy from an identical allocation of wealth and endowments. This is consistent with the ergodic distribution of the model, which implies that on average the two countries consume an identical share of world resources because of symmetry.

We make three remarks. First,  $S_t$  is a key driver of the share of world consumption allocated to the home country,  $SWC_t$ ,

$$SWC_t = \frac{x_t^h + p_t y_t^h}{X_t + p_t Y_t} = \frac{S_t}{1 + S_t}. \quad (3.13)$$

The higher  $S_t$  is, the larger is the home country. Second, as in Colacito and Croce (2013), when the home country receives good news for the endowment of good  $X$ , there is a persistent reduction in the domestic share of world consumption. This counter-cyclical adjustment is consistent with equation (3.12): as good news for the supply of good  $X$  relative to good  $Y$  materializes, the home country experiences a drop in its marginal utility. Therefore, it is optimal to reallocate resources to the foreign country. In the decentralized economy, the home country optimally substitutes part of its current consumption with exports to its foreign trading partner. Third,  $S_t$  introduces an endogenous time-varying volatility term into consumption growth, since allocations are nonlinear functions of this component. In section 4.3, we discuss the importance of this channel in the context of our explanation of the volatility disconnect anomaly.

### 3.1 Calibration and Solution Method

We report our benchmark calibration in table 4. Panel A refers to parameters that have already been employed in this class of models and are standard in the literature (see, among others, Colacito and Croce (2013), and Bansal and Shaliastovich (2013)).

We set the intertemporal elasticity of substitution to 1.5, as in Colacito and Croce (2013). Because of the presence of volatility risk, we can obtain a volatile stochastic discount factor with a risk aversion coefficient of 7, a value particularly conservative in this literature. The subjective discount factor is chosen so as to keep the average annual risk-free rate close to 1% when possible.

The consumption home bias is set to 0.96, a number that falls in the middle of the range observed for our countries. For example, in our sample the US home bias is 0.95, as imports comprise an average of 5% of US consumption goods (Erceg, Guerrieri, and Gust (2008)). Balta and Delgado (2009) document a stronger consumption home bias for the European countries in our dataset and suggest a value of  $\alpha = 0.97$ . Setting  $\lambda = 0.97$

**Table 4: Calibration**

Description	Parameter	Value
<b>Panel A: Standard Parameters</b>		
Relative Risk Aversion	$\gamma$	7
Intertemporal Elasticity of Substitution	$\psi$	1.50
Subjective Discount Factor	$\delta^4$	0.98
Degree of Home Bias	$\alpha$	0.96
Mean of Endowment Growth	$\mu \cdot 4$	2.00%
Short-Run Risk Volatility	$\sigma \cdot \sqrt{4}$	1.87%
Long-Run Risk Autocorrelation	$\rho^4$	0.953
Relative Long-Run Risk Volatility	$\sigma_z/\sigma$	6.90%
Cross-correlation of Short-Run Shocks	$\rho_X$	00.15
Cross-correlation of Long-Run Shocks	$\rho_z$	00.92
<b>Panel B: Time-Varying Short-Run Risk</b>		
Persistence of Short-Run Volatility	$\rho_\sigma$	0.90 [0.89; 0.93]
Volatility of Short-Run Volatility	$\sigma_{sr}$	0.15 [0.15; 0.16]
Cross-correlation of Short-Run Volatility	$\rho_{\sigma,\sigma^*}$	0.30 [0.13; 0.45]
Short-Run Volatility Correlation with Short-Run Shocks	$\rho_{\sigma,\Delta y}$	-0.12 [-0.15; -0.05]

Notes: All parameters are calibrated at quarterly frequency. In panel B, the entries for the data are from the VAR specified in equations (2.6)–(2.7). Numbers in brackets denote the 95% credible intervals. Data are from the OECD dataset and refer to G-17 countries. The sample spans the post-Bretton Wood period, 1971:q1–2013:q4.

would improve our quantitative results, as it would make our risk-sharing channel even more relevant. We prefer to work with  $\alpha = 0.96$  in order to obtain conservative results.

Annualized average output growth is set to 2%, consistent with the empirical findings in table 1. Unconditional volatilities are calibrated to produce an unconditional output volatility of 1.90%, as in the data. The long-run components are calibrated in the spirit of the international long-run risk literature, as they are both highly persistent and correlated across countries (Colacito and Croce (2013)). Since we set  $\sigma_z/\sigma = 0.07\%$ , the implied consumption growth rate is almost *i.i.d.*, as in the data. Short-run output growth shocks, in contrast, are as poorly cross-country correlated as output growth in our dataset (see table 1).

In table 4, panel B, we report the parameters that govern the volatility process of



short-run shocks, that is, the novel and most important element of our investigation. These parameters are calibrated to be consistent with our empirical results. Specifically, we pick values typically in the middle of the Bayesian 95% credible intervals of the VAR system specified in equations (2.6)–(2.7).

Consistent with our data, volatility shocks are as poorly correlated across countries as short-run growth shocks. We allow for negative within-country correlation between volatility and short-run growth shocks so that higher volatility is associated with economic slowdowns. Conditional volatilities are as persistent as in the data.

Given these parameters, we use perturbation methods to solve our system of equations. We compute an approximation of the third order of our policy functions using the `dynare++` package. As documented in Colacito and Croce (2013), a third-order approximation is required to capture endogenous time-varying volatility due to the adjustments of the pseudo-Pareto weights. All variables included in our `dynare++` code are expressed in log-units.

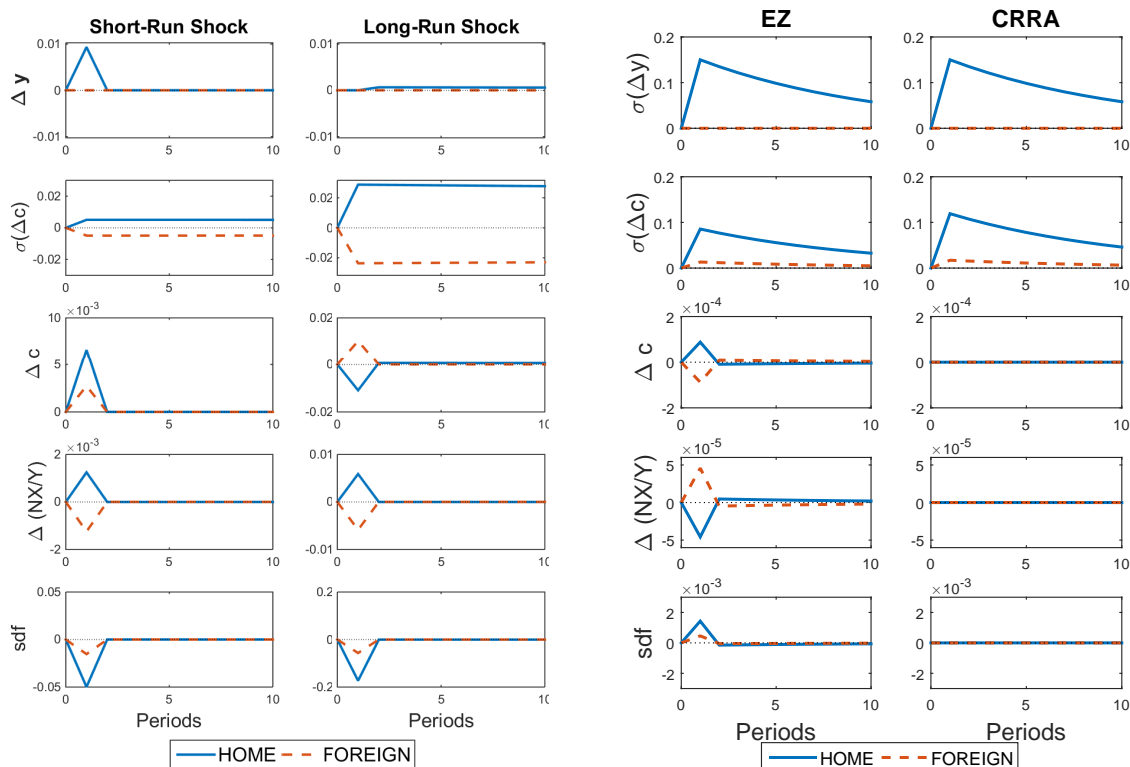
Both the calibration and the solution methods are standard in the literature. In what follows we discuss only the performance of our model for the dynamics of conditional volatilities, that is, the main objective of our investigation. For commonly targeted unconditional moments, we refer the reader to table B1 in the appendix. For the sake of completeness, this table also shows the same moments for the case in which we abstract away from volatility shocks, and for the setting with CRRA preferences.

## 4 Main Results

In this section, we present the main results of our theoretical analysis. We start by describing the risk-sharing motives of both level and volatility shocks. To our knowledge, we are the first to connect recursive risk sharing to evidence on consumption volatility dynamics both within country and in the cross section of countries. We then assess the quantitative performance of our model by means of simulations and show that a frictionless recursive risk-sharing scheme can rationalize our empirical findings.

### 4.1 Risk-Sharing Motives

**Risk sharing of level shocks.** In figure 4(a), we report the response of the variables of interest to a short-run level shock (left panels) and to a long-run level shock (right panels) to the growth rate of the endowment of the home country. Note that on impact the short-run shock is sizeably larger than the long-run shock (figure 4(a), first row of



(a) Level Shocks (EZ only)

(b) Vol Shock (EZ vs CRRA)

**Figure 4 - Impulse Responses.** Panel (a) shows the percentage impulse response functions of output growth ( $\Delta y$ ), consumption growth volatility ( $\sigma(\Delta c)$ ), consumption growth ( $\Delta c$ ), change of net-export-output ratio ( $\Delta NX/Y$ ), and stochastic discount factors ( $sdf$ ) to a shock to the home endowment for both the home country (solid line) and the foreign country (dashed line). Level shocks materialize only in the home country, and only at time 1. Shocks are not orthogonalized; we consider a positive  $\sigma$  shock in the short-run, and a positive  $\sigma_x$  shock for the long-run. In panel (b) we consider an endowment volatility shock which is orthogonalized within and across countries, i.e., it affects only the home country and it does not change the growth rate level. All parameters are calibrated to the quarterly values reported in Table 4.

panels). However, the long-run shock is highly persistent, and it ultimately affects the growth rate of the home endowment for a large number of periods.

Consistent with Colacito and Croce (2013), the growth rates of consumption increase in both countries in response to a positive short-run shock, whereas they move in opposite directions in response to a positive long-run shock (figure 4(a), third row of panels). The asymmetric response of consumption growth rates to a long-run endowment shock is the result of the agents' extreme sensitivity to persistent news to the growth rates of their endowments.

When a shock of this nature materializes, the home country's marginal utility drops substantially (figure 4(a), bottom-right panel). To restore the equality of the marginal utilities of consumption across countries, an international redistribution of resources must take place. Specifically, the home country increases its exports, while the foreign country increases its imports (figure 4(a), fourth row of panels). Equivalently, the ratio of the pseudo-Pareto weights  $S_t$  declines, as dictated by equation (3.12).

Since the long-run shock is a pure news shock, that is, a shock that results in a larger amount of home endowment only in future time periods, the international redistribution of resources takes place through a drop in home consumption and an increase in foreign consumption. As pointed out in Colacito and Croce (2013), this immediate response of the consumption level simultaneously comes with an opposite swap of long-run consumption variance (as measured by  $\sigma_t(U_{t+1})$ ). Specifically, the home country optimally reduces its current consumption share,  $S_t/(1 - S_t)$ , in exchange for a reduction in  $\sigma_t(U_{t+1})$  (see Figure C1, top-right panel). Consistent with equation (3.3), the reduction of long-term uncertainty improves welfare.

**Risk sharing of vol shocks.** Figure 4(b) shows the response of our main set of variables of interest to a volatility shock in the home country. For comparability, we report the responses from both our benchmark model and a model with standard time-additive CRRA preferences.

We first point out that the responses of consumption, net exports, and stochastic discount factors in the model with EZ preferences are the mirror image of those obtained for a positive long-run endowment shock, since a positive volatility shock is a negative news shock.

Second, we note that the relative response of the volatilities of consumption growth rates in the two countries differs across the two preference specifications. With CRRA preferences, volatility news shocks are not directly priced and hence marginal utilities do not move. There is no reallocation of resources across countries, and as a result the increase in volatility of the domestic endowment is almost entirely absorbed by domestic consumption. According to our definition of volatility pass-through, in this situation our index takes on a value close to zero.

In the specification with EZ preferences, the risk-sharing motive that we described in the previous section partially offsets the increased amount of volatility in the economy. To represent the effects of volatility shocks in the model, we consider volatility frontiers which depict the equilibrium volatility-related quantities as a function of the Pareto weight  $S_t$ . In figure 5(a), we show the change in consumption volatility following

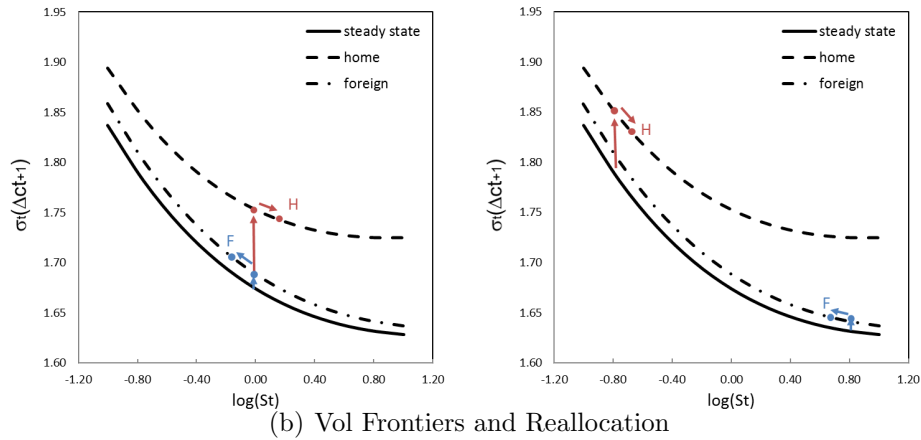
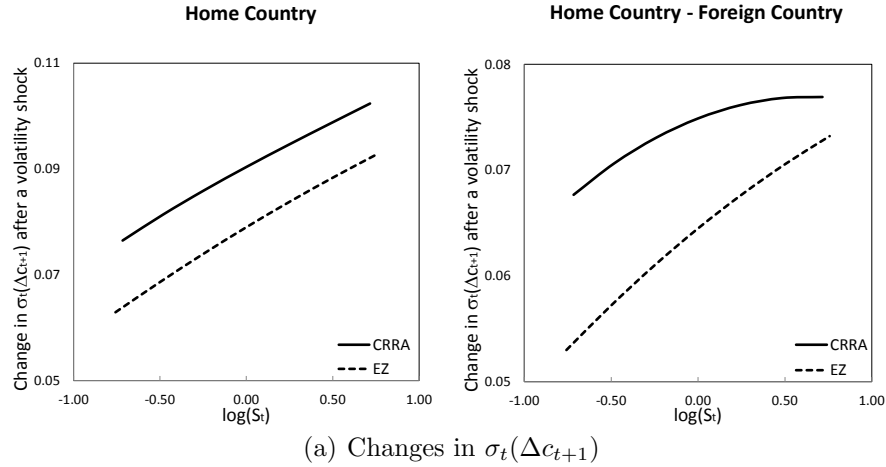
the adverse output volatility shocks at home (left panel), and home relative to abroad (right panel). In figure 5(b), we consider reallocation effects, and show the equilibrium consumption volatility, as a function of the size, at home and abroad. As shown in the left panel, upon the arrival of an adverse volatility shock, the short-run consumption volatility frontier of the home country shifts upward by a lesser extent than under CRRA preferences. Since our agents are averse to conditional variance, their trade is arranged to reduce the time variation of their own conditional volatilities. Furthermore, these countries tend to keep their volatilities aligned to each other. As shown in the right panel of figure 5(a), the cross-country difference of the conditional volatilities increases by a smaller amount under EZ preferences than in the CRRA setting. We address this finding in further detail in our discussion of our volatility pass-through results below.

## 4.2 Volatility Comovements and Pass-through

**Unconditional comovements.** We use simulations to quantify the ability of the model to reproduce our empirical findings for the dynamics of volatility. We find it useful to consider first a setting with standard CRRA preferences, as in this case shocks do not produce any sizeable endogenous reallocation of volatility from one country to the other. Equivalently, the pseudo-Pareto weights are almost constant (Colacito and Croce (2013)), similarly to the autarky scenario in Cole and Obstfeld (1991). In this situation, the correlation between consumption and output volatility within each country is almost perfect, as the volatility of consumption moves one-to-one with the volatility of the output growth rate.

In contrast to the CRRA case, our model with recursive preferences is able to produce a less-than-perfect contemporaneous correlation between output and consumption volatility. This result is driven by the fact that level shocks are an important endogenous driver of consumption volatility independently of our exogenous output volatility shocks. As shown in table 5, without volatility shocks, the cross-country correlation of the consumption profiles would be almost perfectly negative. This channel counterbalances the tendency for consumption profiles to be more correlated than output. At the equilibrium, our model produces a final correlation of 35%, a figure that is well within the confidence region of our cross section of countries.

The international correlation of the consumption volatilities is 50%, a number slightly higher than that observed in the data. Recall that the exogenous international correla-



**Figure 5 - Response to an Adverse Volatility Shock to Good  $X$ .** In figure (a), the left panel reports the change in the conditional volatilities of consumption growth in the home country after an adverse shock to the volatility of the good  $X$ . The right panel reports the change in the cross-country difference of conditional volatility of consumption growth for the same shock. In each panel, the dashed (solid) line refers to the case of EZ (CRRA) preferences. Across all cases, we keep all other exogenous state variables fixed at their unconditional mean. In figure (b), we depict the equilibrium conditional volatility of the consumption growth of the home country. The solid line refers to the frontier at the steady state. The dashed lines show the shift of the frontier after an adverse volatility shock of the good  $X$ . The log of the relative size of the home country is denoted by  $\log(S)$ .

tion of the output volatility shocks is set to 30%. Since the pseudo-Pareto weights are almost fixed under CRRA, the log-consumption bundles in each country are a constant weighted average of the two goods, and hence their volatilities are more correlated than those of the two underlying endowment processes.

**Table 5: Comovements and Pass-Through**

<b>Panel A: Unconditional comovements</b>					
	Avg.	Quintiles [ 1 <sup>st</sup> ; 4 <sup>th</sup> ]	Bench- mark	No TVV ( $\sigma_\sigma = 0$ )	CRRA ( $\gamma = 7$ )
$corr(\sigma_t(\Delta c_{t+1}), \sigma_t(\Delta y_{t+1}))$	0.65	[0.26; 0.80]	0.88	–	0.98
$corr(\sigma_t(\Delta c_{t+1}), \sigma_t(\Delta c_{t+1}^*))$	0.45	[0.35; 0.66]	0.35	-0.93	0.50
<b>Panel B: Pass-through and size</b>					
	SWC	US vol shock	Foreign vol shock		
<i>US/G7 Countries:</i>					
Data	[0.44; 0.51]	[0.43; 0.54]	[0.51; 0.63]		
Model (EZ)	0.50	0.53	0.53		
Model (CRRA)	0.50	0.30	0.30		
<i>US/Bottom-10 G17 Countries</i>					
Data	[0.72; 0.77]	[0.45; 0.57]	[0.66; 0.78]		
Model (EZ)	0.72	0.39	0.70		
Model (CRRA)	0.72	0.38	0.37		

Notes: In panel A, we report correlations between the conditional volatility ( $\sigma_t$ ) of consumption and output growth within and across countries. Conditional volatilities are obtained by estimating equation (2.1) country by country. The data refer to G-17 countries and are described in section 2.1. Panel B reports estimated pass-through coefficients (see equation (2.4)) with respect to both domestic (US) and foreign volatility shocks for both the G7 and bottom-10 G17 countries. For each country, we compute the moments of interest over the post-Bretton Wood period, 1971:Q1–2013:Q4. For each moment, we report first and fourth cross-country quintiles. The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 4.

**Pass-through.** Overall, unconditional comovements do not allow us to discriminate between the CRRA and EZ settings, as both models produce results that lie within the empirical ranges. This conclusion changes when we focus on conditional responses and, in particular, on our pass-through index. When we compare countries of similar size, that is, the US versus the remaining G7 countries, only the model with EZ preferences generates a pass-through of 50%, as in the data. This result is particularly relevant because it is obtained with a simultaneous response of the current account that replicates that observed in the data, as shown in figure 2. Under CRRA preferences, however, the pass-through is very limited, as volatility news shocks are not an independent determinant of risk-sharing motives.

Furthermore, when we alter the relative consumption share in the model and set the home country consumption to be about three times larger than that of the foreign country, as in the comparison between the US and the bottom-10 G17 countries, our model can replicate the asymmetry documented in the data. Specifically, our model

predicts that when a volatility shock hits a big country, the pass-through is limited. Vice versa, small countries can better share shocks to their endowment volatility as documented by their higher pass-through.

Figure 5(b) helps us in understanding this result as it shows that the consumption volatility frontier is downward sloping and convex.<sup>8</sup> When an adverse volatility shock affects good  $X$ , the volatility frontier shifts upward both for the home and the foreign country. Because of home bias, however, the shift is stronger for the home country. Under the optimal risk sharing scheme, the home country receives a positive transfer and becomes larger, and hence faces less consumption growth volatility going forward, whereas the opposite applies to the foreign country. This adjustment keeps the consumption volatility of the two countries closer to each other and makes our pass-through index sizeable. When the home country has a share of world consumption of about 30%, this phenomenon is more pronounced and makes the pass-through stronger (figure 5(b), right panel). In the CRRA case, however, the relative country size does not play any major role in determining the extent of volatility pass-through, a result that is at odds with the data.

**Financial pass-through.** To compute our financial pass-through index in the model, we compute the equilibrium returns of a claim to a dividend cash-flow specified as follows:

$$\Delta d_t = \mu + k(\Delta \log X_t - \mu) + \sigma_d \epsilon_{d,t}, \quad \Delta d_{t+1}^* = \mu + k(\Delta \log Y_t - \mu) + \sigma_d \epsilon_{d,t}^*,$$

which is a levered claim to local output plus a pure local cash-flow shock.<sup>9</sup> We set  $k = 2$  and  $\sigma_d = 8.5\%$  in order to have an equilibrium equity premium under complete markets of 6% and a Sharpe-ratio of 0.30.

As shown in figure 2 (bottom-right panel), our benchmark model tracks very well the response of the returns volatility differential upon the arrival of an output volatility shock. As a result, this model is able to deliver a financial pass-through of 0.87, which is very close to the data.

Our empirical evidence suggests that volatility pass-through has increased over time (see figure 3). In order to address this fact, we consider the hypothesis that this time-variation has been driven by increased financial integration. As in Cole and Obstfeld

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<sup>8</sup>As we explain in detail in Appendix C, the shape of the frontier follows directly from the properties of our aggregator across goods. Since the Inada conditions apply, as a country becomes small, the sensitivity of its consumption bundle to transfers of resources increases. As a result, our risk sharing-driven reallocation increases the volatility of the consumption bundle for small countries. This mechanism is more pronounced as size approaches zero, i.e., it makes the consumption volatility frontier convex.

<sup>9</sup>As in Bansal and Yaron (2004),  $\epsilon_d$  is uncorrelated to all other fundamental shocks and is not priced.

**Table 6: Change in Pass-Through Indexes**

	Benchmark	CRRA
Consumption vol pass-through	0.40	0.20
Financial vol pass-through	0.57	0.00

Notes: The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 4. We report the difference between the pass-through index under complete markets and financial autarky setting in both cases SWC=50%.

(1991), we focus on two extreme regimes: one in which markets are complete, and one in which each country is in financial autarky ( $A_t^h = A_t^f = 0 \forall t$ ). The equilibrium under financial autarky can be computed by replacing equation (3.12) with  $S_t = 1 \forall t$ .

As shown in figure 2 (bottom-right panel), our benchmark model tracks very well the response of the returns volatility differential upon the arrival of an output volatility shock. As a result, this model is able to deliver a financial pass-through of 0.87, which is very close to the data. Most importantly, the model with recursive preferences predicts that the volatility pass-through should increase substantially with financial integration both for consumption and equity returns (Table 6). With time-additive preferences, volatility shocks are not directly priced and hence financial integration cannot produce any change in the extent of financial pass-through.

### 4.3 Risk Sharing and the Volatility Disconnect Anomaly

In table 7, we compare our empirical findings on the disconnect between exchange rates and consumption differentials to our simulation results. In the top panel, we show that our benchmark model is able to replicate the slightly negative correlation between consumption growth differentials and exchange rate movements observed in the data over both a quarterly and an annual horizon. As in the model with constant volatility (Colacito and Croce (2013)), news shocks are sufficient to break the perfect correlation of the consumption differentials and the exchange rate. Consistent with the observation in Backus and Smith (1993), under CRRA preferences this correlation is counterfactually high.

The model with the CRRA preferences also delivers a perfect positive correlation between the conditional variances of consumption differentials and the exchange rate (bottom portion of table 7, rightmost column). Interestingly, this correlation switches to large and negative in the recursive utility model without time-varying volatilities (our ‘No TVV’ case), which is the model analyzed by Colacito and Croce (2013). The



**Table 7: Volatility Disconnect Anomaly and Risk Sharing**

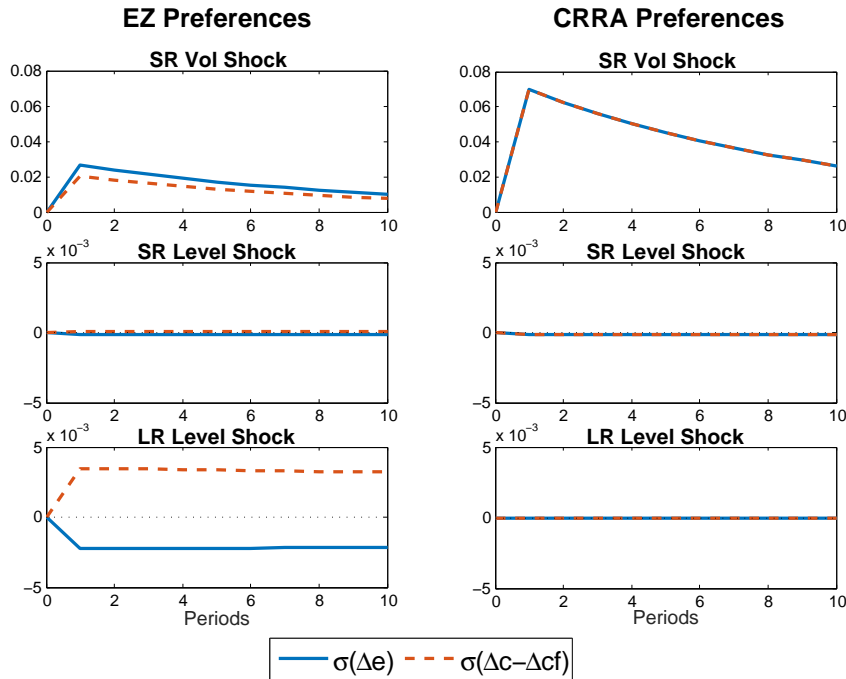
	G-17 Data		Model		
	Aver.	Quintiles [ 1 <sup>st</sup> ; 4 <sup>th</sup> ]	Bench- mark	No TVV ( $\sigma_\sigma = 0$ )	CRRA ( $\gamma = 7$ )
<i>Levels Disconnect</i>					
$corr(\Delta cd_{t+1}, \Delta e_{t+1})$	-0.13	[ -0.19; -0.04 ]	-0.25	-0.27	1.00
$corr(\Delta \widehat{cd}_{t+4}, \Delta \widehat{e}_{t+4})$	-0.14	[ -0.29; -0.05 ]	-0.21	-0.24	1.00
<i>Volatility Disconnect</i>					
$corr(\sigma_t(\Delta cd_{t+1}), \sigma_t(\Delta e_{t+1}))$	0.20	[ -0.01; 0.42 ]	0.56	-0.75	1.00
$corr(\sigma_t(\Delta \widehat{cd}_{t+4}), \sigma_t(\Delta \widehat{e}_{t+4}))$	0.26	[ -0.02; 0.52 ]	0.47	-0.75	1.00

Notes: This table reports key moments for real consumption growth differentials ( $\Delta cd = \Delta c - \Delta c^*$ ) and exchange rate growth ( $\Delta e$ ). Foreign variables are marked by ‘\*’; cumulative growth rates are denoted by ‘^’. Conditional log-volatilities are denoted by  $\sigma_t$ . The empirical moments are obtained by estimating equation (2.1) country by country, as detailed in section 2.2. The data refer to G-17 countries and are described in section 2.1. For each country, we compute the moments of interest over the post-Bretton Wood period, 1971:Q1–2013:Q4, as detailed in section 2.1. For each moment, we report (i) its GDP-weighted average across countries; and (ii) its first and fourth cross-country quintiles. The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 4.

predictions of both of these restricted models are at odds with the data: the empirical estimates suggest a positive but weak correlation of about 20-30%. Our full model, on the other hand, delivers a positive and mild correlation of about 50%, which is closer to the data. These findings highlight the role of the recursive utility and output volatility shocks to resolve the volatility disconnect anomaly. To explain the economic mechanisms behind the results, we consider separate impact of volatility and level shocks on the conditional variances of consumption differential and the exchange rates. These responses are depicted in figure 6.

**Volatility shock.** A volatility shock in the home country produces a *positive* comovement between the volatility of the exchange rate and that of the differential of consumption growth rates. This is because the two countries share the risk associated with an increase in macroeconomic uncertainty, as explained in the previous section. Hence, in the absence of level shocks, we would have a perfect connection between exchange rate and consumption differential volatility. This is true both in the recursive utility and the CRRA model.

**Short-run shocks.** We note that short-run shocks are irrelevant in this context, as



**Figure 6 - Impulse Response Functions and Volatility Disconnect.** This figure shows the percentage response of the volatility of consumption growth differentials (dashed line) and exchange rate growth rate volatility (thick line) to a volatility shock in the home country (top panels), a short-run shock in the home country (middle panels), and a long-run shock in the home country (bottom panels). The left (right) panels report the response functions for our benchmark model with EZ (CRRA) preferences.

they result in a negligible response of the two volatilities, since investors' marginal utilities are not particularly sensitive to this type of shock (figure 6, middle-left panel).

**Long-run shocks.** In contrast to short-run shocks, in a recursive-utility environment a long-run shock to the home country generates a significant *negative* comovement between the two volatilities and lowers their unconditional correlation (figure 6, bottom-left panel). Over an annual horizon, this channel enables our model to produce a correlation within our empirical range (table 7, bottom two lines). In a model with CRRA preferences, long-run shocks have no impact on the two conditional volatilities. Hence, all the effect is driven by volatility shocks, which leads to a perfect positive correlation between the conditional volatilities of consumption differential and the exchange rates.

To explain the origin of this negative comovement, it is useful to decompose the

variance of the consumption differential growth rate into its subcomponents:

$$\begin{aligned} \text{Var}_t(\Delta c_{t+1} - \Delta c_{t+1}^*) &= \text{Var}_t(\Delta c_{t+1}) + \text{Var}_t(\Delta c_{t+1}^*) \\ &\quad - 2 \cdot \sqrt{\text{Var}_t(\Delta c_{t+1}) \cdot \text{Var}_t(\Delta c_{t+1}^*)} \cdot \text{corr}_t(\Delta c_{t+1}, \Delta c_{t+1}^*). \end{aligned} \quad (4.1)$$

At the equilibrium, the conditional correlation of consumption growth rates is almost time invariant.<sup>10</sup> As a result, the dynamics of the variance of consumption differentials is mostly determined by the sum of the variances of the consumption growth rates across countries, as depicted in the left panel of figure 7.

Because of the convexity of the short-run volatility frontier (Figure 5(b)), the sum of the variances of the growth rates of consumption is increasing in wealth inequality, that is, it is U-shaped with respect to the log-ratio of the Pareto weights (figure 7, left panel). As a result, starting from an equal distribution of wealth,  $\sigma_t(\Delta c_{t+1} - \Delta c_{t+1}^*)$  increases upon the arrival of a long-run shock (figure 6, bottom-left panel).

Given our assumption of complete markets, the variance of the exchange rate growth can be decomposed as follows:

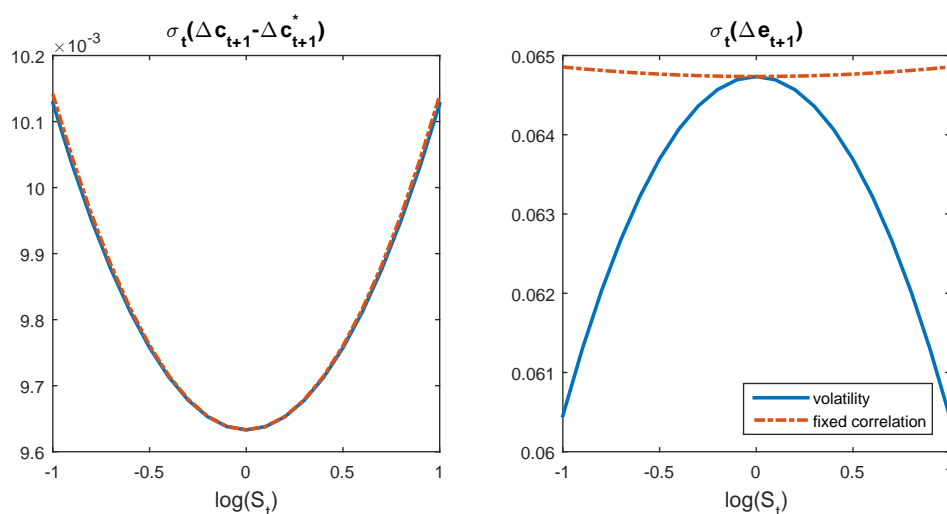
$$\begin{aligned} \text{Var}_t(\Delta e_{t+1}) &= \text{Var}_t(\Delta m_{t+1} - \Delta m_{t+1}^*) = \text{Var}_t(\Delta m_{t+1}) + \text{Var}_t(\Delta m_{t+1}^*) \\ &\quad - 2\sqrt{\text{Var}_t(\Delta m_{t+1}) \cdot \text{Var}_t(\Delta m_{t+1}^*)} \cdot \text{corr}_t(\Delta m_{t+1}, \Delta m_{t+1}^*). \end{aligned}$$

In a model with long-run growth news, most of the volatility of the stochastic discount rates is driven by the continuation utilities. In Appendix C (top-right panel of figure C1), we show that the utility variance frontier is linear, meaning that the drop in the conditional volatility of the utility of one country is almost entirely offset by the increase in volatility of the other country. As a result,  $\text{Var}_t(\Delta m_{t+1}) + \text{Var}_t(\Delta m_{t+1}^*)$  is close to being time invariant and the conditional volatility of the exchange rate is mostly explained by the endogenous time variation in the correlation of the stochastic discount factors (figure 7, right panel).

With recursive preferences, the reallocation prompted by long-run shocks keeps the continuation utilities of the two agents aligned to each other, that is, it introduces a positive cross-country comovement of continuation utilities and hence stochastic discount factors.<sup>11</sup> Because our utility function satisfies the Inada's conditions, the strength of the reallocation channel is enhanced when one of the two countries is small. Equivalently,

<sup>10</sup>This correlation is driven by the positive comovement between the short-run shock of a country and the adjustment in the share of consumption of the other country. In equilibrium, this correlation increases modestly in wealth inequality.

<sup>11</sup>When a country receives good news for the long run, its utility increases immediately, reflecting the



**Figure 7 - Conditional Volatilities Disconnect.** The left panel plots the conditional volatility of the difference between the growth rate of consumption in the home and foreign countries,  $\sigma_t(\Delta c_{t+1} - \Delta c_{t+1}^*)$ . The right panel depicts the conditional volatility of the growth rate of the exchange rate,  $\sigma_t(\Delta e_{t+1})$ . Both volatilities are plotted against the logarithm of the ratio of the pseudo-Pareto weights,  $S_t$ . Across all cases, both the exogenous long-run components and the exogenous volatility processes are fixed at their unconditional mean. In each panel, the solid line refers to the conditional volatility obtained at the equilibrium, whereas the dashed line refers to the conditional volatility obtained by holding the correlations fixed at their unconditional mean in equations (4.1)–(4.2).

the correlation of the stochastic discount factors increases with wealth inequality. As a result, the exchange rate volatility has an inverse U-shape with respect to the log-ratio of the Pareto weights (see the right panel of figure 7). Thus starting from an equal distribution of wealth, the impulse response of the exchange rate volatility is negative, in sharp contrast to the response of the volatility of the consumption differentials.

Without volatility shocks (Colacito and Croce, 2013), the endogenous response of volatilities to long-run shocks dominates and results in a counterfactual negative correlation between exchange rate and consumption differential conditional volatility. Our recursive risk-sharing of volatility shocks overcomes this problem, but it is admittedly unable to reproduce the full disconnect observed in the data.<sup>12</sup>

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total discounted impact of the news. The other country benefits from the international redistribution of resources, which determines an increase in its share of consumption. Given the persistent nature of the consumption shares, the other country also experiences an increase in the present value of its consumption and, thus, its utility. As a consequence, the extent of comovement of the continuation utilities (and of the stochastic discount factors in general) increases.

<sup>12</sup>In the data, the attenuation bias may result in a lower estimated value for the correlation of interest,

**Welfare Implications.** We conclude this section by assessing the relevance of volatility shocks for welfare. Specifically, we compare the average welfare under both our benchmark model and that without uncertainty shocks (No TVV case) and find that each agent would be willing to give up to 2.58% of life-time consumption in order to avoid uncertainty shocks.<sup>13</sup> This figure confirms that uncertainty shocks are relevant determinants of risk.

## 5 Conclusion

In this paper, we construct a measure of bilateral volatility pass-through and we use it to document the sizeable extent of international propagation of output volatility shocks. Furthermore, we provide novel empirical evidence regarding the disconnect between the volatility of consumption differentials and the volatility of exchange rates. We show that these findings constitute a puzzle from the standpoint of a frictionless model with CRRA preferences. We then develop a frictionless general equilibrium model featuring long-run growth news shocks, volatility shocks, and two countries populated by agents with recursive preferences and demonstrate that our model can replicate these empirical findings.

Future developments should focus on extending this setting to international real business cycle models in an effort to better understand the role of international investment flows and international frictions in the origination and international propagation of volatility shocks. The investigation of the roles of trading frictions, portfolio composition, and market incompleteness are other promising directions for future research.

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since volatilities are measured with error. This could partially explain the gap between this correlation in our benchmark model and its empirical counterpart. We thank Tarek Hassan for pointing this out.

<sup>13</sup>Since our two countries are calibrated in a symmetric way, this number applies to both of them.

## References

- Anderson, E., (2005), The dynamics of risk-sensitive allocations, *Journal of Economic Theory* 125(2), 93–150.
- Backus, D. K. and G. Smith, (1993), Consumption and real exchange rates in dynamic economies with non-traded goods, *Journal of International Economics* 35, 297–316.
- Bakshi, G., M. Cerrato, and J. Crosby, (2017), Implications of incomplete markets for international economies, *Review of Financial Studies* .
- Bansal, R., D. Kiku, I. Shaliastovich, and A. Yaron, (2014), Volatility, the macroeconomy, and asset prices, *Journal of Finance* 69, 2471–511.
- Bansal, R. and I. Shaliastovich, (2013), A long-run risks explanation of predictability puzzles in bond and currency markets, *Review of Financial Studies* 26, 1–33.
- Bansal, R. and A. Yaron, (2004), Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Berg, K. A. and N. C. Mark, (2018), Global macro risks in currency excess returns, *Journal of Empirical Finance* .
- Bloom, N., (2009), The impact of uncertainty shocks, *Econometrica* 77, 623–89.
- Chernov, M., J. Graveline, and I. Zviadadze, (2018), Crash risk in currency returns, *Journal of Financial and Quantitative Analysis* .
- Cogley, T. and T. J. Sargent, (2005), Drifts and volatilities: monetary policies and outcomes in the post-WWII US, *Review of Economic Dynamics* 8, 262–302.
- Colacito, R., (2008), Six anomalies looking for a model. a consumption based explanation of international finance puzzles, Working paper, University of North Carolina, Chapel Hill.
- Colacito, R. and M. M. Croce, (2013), International asset pricing with recursive preferences, *Journal of Finance* 68, 2651–86.
- Cole, H. and M. Obstfeld, (1991), Commodity trade and international risk sharing: How much do financial markets matter?, *Journal of Monetary Economics* 28, 3–24.
- Della Corte, P., S. J. Riddiough, and L. Sarno, (2016), Currency Premia and Global Imbalances, *The Review of Financial Studies* 29.
- Della Corte, P., L. Sarno, and I. Tsiakas, (2009), An Economic Evaluation of Empirical Exchange Rate Models, *Review of Financial Studies* 22, 2491–530.
- Della Corte, P., L. Sarno, and I. Tsiakas, (2011), Spot and forward volatility in foreign exchange, *Journal of Financial Economics* 100, 496 – 513.
- Du, W., C. Pflueger, and J. Schreger, (2017), Sovereign debt portfolios, bond risks, and the credibility of monetary policy, *Working Paper* .
- Epstein, L. G. and S. Zin, (1991), Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical analysis, *Journal of Political Economy* 99, 263–86.

- Erceg, C., L. Guerrieri, and C. Gust, (2008), Trade Adjustment and the Composition of Trade, *Journal of Economic Dynamics and Control* 32, 2622–50.
- Farhi, E., S. Fraiberger, X. Gabaix, R. Ranciere, and A. Verdelhan, (2015), Crash risk in currency markets, Working paper.
- Farhi, E. and X. Gabaix, (2016), Rare disasters and exchange rates, *Quarterly Journal of Economics* 131(1), 1–52.
- Gabaix, X. and M. Maggiori, (2015), International liquidity and exchange rate dynamics, *Quarterly Journal of Economics* 130, 1369–420.
- Gavazzoni, F., B. Sambalaibat, and C. Telmer, (2013), Currency risk and pricing kernel volatility, Working paper.
- Gilchrist, S., J. Sim, and E. Zakrajsek, (2014), Uncertainty, financial frictions and investment dynamics, Working paper, NBER.
- Gourinchas, P. O. and H. Rey, (2007), International financial adjustment, *Journal of Political Economy* 115, 665–703.
- Gourio, F., M. Siemer, and A. Verdelhan, (2014), Uncertainty and international capital flows, Working paper.
- Hassan, T., (2013), Country size, currency unions, and international asset returns, *The Journal of Finance* 68, 2269–308.
- Hassan, T., T. Mertens, and T. Zhang, (2015), Currency manipulation, *Working Paper* .
- Hassan, T. A., T. M. Mertens, and T. Zhang, (2016), Not so disconnected: Exchange rates and the capital stock, *Journal of International Economics* 99, S43–S57.
- Heyerdahl-Larsen, C., (2015), Asset Prices and Real Exchange Rates with Deep Habits., *Review of Financial Studies* 27, 3280–317.
- Jones, L., R. Manuelli, H. Siu, and E. Stacchetti, (2005), Fluctuations in convex models of endogenous growth I: Growth effects, *Review of Economic Dynamics* 8, 780–804.
- Jurado, K., S. C. Ludvigson, and S. Ng, (2015), Measuring uncertainty, *American Economic Review* 105, 1177–216.
- Justiniano, A. and G. E. Primiceri, (2008), The time-varying volatility of macroeconomic fluctuations, *American Economic Review* 98, 604–41.
- Kim, S., N. Shephard, and S. Chib, (1998), Stochastic volatility: Likelihood inference and comparison with ARCH models, *Review of Economic Studies* 65, 361–93.
- Kollmann, R., (1991), Essays on international business cycles, *University of Chicago* .
- Kollmann, R., (2016), International business cycles and risk sharing with uncertainty shocks and recursive preferences, *Journal of Economic Dynamics and Control* forthcoming.
- Lettau, M., S. Ludvigson, and J. Wachter, (2008), The declining equity premium: What role does macroeconomic risk play?, *Review of Financial Studies* 21, 1653–87.

- Lettau, M., M. Maggiori, and M. Weber, (2014), Conditional risk premia in currency markets and other asset classes, *Journal of Financial Economics* 114, 197 – 225.
- Lewis, K., (2011), Global Asset Pricing, *Annual Reviews of Financial Economics* 3.
- Lustig, H., N. Roussanov, and A. Verdelhan, (2011), Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–77.
- Lustig, H., N. Roussanov, and A. Verdelhan, (2014), Countercyclical currency risk premia, *Journal of Financial Economics* 111(3), 527–553.
- Lustig, H. and A. Verdelhan, (2007), The cross section of foreign currency risk premia and consumption growth risk, *American Economic Review* 97, 89–117.
- Lustig, H. and A. Verdelhan, (2018), Does incomplete spanning in international financial markets help to explain exchange rates?, *Working Paper* .
- Maggiori, M., (2017), Financial intermediation, international risk sharing, and reserve currencies, *American Economic Review* .
- Maggiori, M., B. Neiman, and J. Schreger, (2017), International currencies and capital allocation, *Working Paper* .
- McConnell, M. and G. P. Quiros, (2000), Output fluctuations in the united states: What has changed since the early 1980s?, *American Economic Review* 90, 1464–76.
- Mueller, P., A. Stathopoulos, and A. Vedolin, (2017), International correlation risk, *Journal of Financial Economics* .
- Obstfeld, M., (1998), The global capital market: Benefactor or menace?, *Journal of Economic Perspectives* 12, 9–30.
- Pavlova, A. and R. Rigobon, (2007), Asset prices and exchange rates, *Review of Financial Studies* 20, 1139–1181.
- Pavlova, A. and R. Rigobon, (2010), An asset-pricing view of external adjustment, *Journal of International Economics* 80, 144–156.
- Pavlova, A. and R. Rigobon, (2013), International macro-finance, *Handbook of Safeguarding Global Financial Stability: Political, Social, Cultural, and Economic Theories and Models* 2, 169–176.
- Primiceri, G. E., (2005), Time varying structural vector autoregressions and monetary policy, *Review of Economic Dynamics* 72, 821–52.
- Quinn, D. P., (1997), The correlates of change in international financial regulation, *American Political Science Review* 91(3), 531–551.
- Quinn, D. P. and H.-J. Voth, (2008), A century of global equity markets correlations, *American Economic Review* 98, 535–40.
- Ramey, G. and V. Ramey, (1995), Cross-country evidence on the link between volatility and growth, *American Economic Review* 85, 1138–51.



- Ready, R., N. Roussanov, and C. Ward, (2017), Commodity trade and the carry trade: A tale of two countries, *Journal of Finance* .
- Sandulescu, M., F. Trojani, and A. Vedolin, (2017), Model-free international sdfs in incomplete markets, *Working Paper, University of Geneva, University of Lugano, and Boston University* .
- Stathopoulos, A., (2017), Asset prices and risk sharing in open economies, *Review of Financial Studies* .
- Stock, J. and M. Watson, (2002), Has the business cycle changed and why, in Gertler, M. and K. Rogoff, eds., *NBER Macroeconomics Annual: 2002*, MIT Press, Cambridge, MA.
- Taylor, A. M., (2002), A century of current account dynamics, *Journal of International Money and Finance* 21, 725–48.
- Verdelhan, A., (2010), A habit-based explanation of the exchange rate risk premium, *The Journal of Finance* 65(1), 123–45.
- Zviadadze, I., (2017), Term structure of consumption risk premia in the cross section of currency returns, *Journal of Finance* .

# Appendix A. Volatility Estimation

We use an auxiliary mixture sampler to estimate the model specified in (2.1) and extract latent volatility components, following Kim, Shephard, and Chib (1998). Specifically, we rewrite the observation equation,

$$\log((z_t - \mu - \rho z_{t-1})^2) = \sigma_t + \log(\eta_t^2). \quad (\text{A1})$$

The distribution of  $\log(\eta_t^2)$  can be well approximated by a mixture of Gaussian distributions:

$$p(\log(\eta_t^2)) = \sum_{i=1}^n \pi_i \varphi(\eta_t; \mu_{\eta,i}, \sigma_{\eta,i}^2), \quad (\text{A2})$$

where  $\varphi$  is the probability density function of a Gaussian distribution with mean  $\mu_{\eta,i}$  and standard deviation  $\sigma_{\eta,i}$ . In the Markov Chain Monte Carlo procedure,  $s_t \in [1, T]$  is drawn to indicate one Gaussian distribution to sample  $\log(\eta_t^2)$ . Conditioning on  $s_t$ , the model is in Gaussian linear state-space form, and a standard forward-filtering, backward-sampling scheme can be applied. The algorithm thus takes the form:

1. Initialize  $\mu, \rho, \mu_\sigma, \nu, \sigma_\omega, s_t$
2. Sample  $\sigma_t$  from  $p(\sigma_t | z, \mu, \rho, \mu_\sigma, \nu, \sigma_\omega, s_t, z)$
3. Sample  $s_t$  from  $p(s_t = i) \propto \pi_i \varphi(\log((z_t - \mu - \rho z_{t-1})^2); \sigma_t + \mu_{\eta,i}, \sigma_{\eta,i}^2)$
4. Sample  $\mu, \rho, \mu_\sigma, \nu, \sigma_\omega$  from  $p(\mu, \rho, \mu_\sigma, \nu, \sigma_\omega | \sigma_t, z)$
5. Repeat 2–4 until convergence

In our empirical implementation the priors are very loose:  $\mu \sim N(0.005, 2)$ ,  $\rho \sim N(0.3, 100)$ ,  $\mu_\sigma \sim N(-10, 20)$ ,  $\nu \sim N(0.9, 0.5)$ , and  $\sigma_\omega \sim IG(2, 0.5)$ . We sample 20,000 times and discard the first 5,000. The posterior mean of  $\sigma_t$  is the volatility used in the empirical analysis.

## Appendix A.1. Robustness of Empirical Results

In this section, we verify that our key empirical evidence on the volatility risk sharing is robust to several modifications of our benchmark analysis. In table A1, Panel A, we show that our results remain unchanged when we let the volatility processes be the ‘least primitive’ processes in the context of the Cholesky decomposition, by replacing equations (2.3) and (2.7) with

$$\tilde{Y}_{i,t} = \begin{bmatrix} \Delta y_i - \Delta y_{US} \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \\ \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \end{bmatrix}, \quad (\text{A3})$$

and

$$Y'_{i,t} = \begin{bmatrix} \Delta y_i & \Delta y_{US} & \sigma_t(\Delta y_i) & \sigma_t(\Delta y_{US}) & \sigma_t(\Delta c_i) & \sigma_t(\Delta c_{US}) \end{bmatrix}, \quad (\text{A4})$$

respectively. We have tested other orders as well, with similar results. We omit them for the sake of brevity.

In Panel B, we re-estimate the volatility processes in equation (2.1) and account for the potential correlation of level shocks,  $\eta_t$ , and volatility shocks,  $\omega_t$ . The sampling of the volatility parameters follows an acceptance-rejection Metropolis-Hastings algorithm. In Panel C, we re-estimate the volatility processes by including the lag of output growth, consumption growth, and the net-exports-to-output ratio in the set of our forecasting variables used in the first equation in (2.1). In Panel D, we assess our VAR results by looking at the US against the remaining G7/Bottom10 countries, assuming that they all share the same parameters of the system of equations (2.1). In Panel E, we run our benchmark empirical estimation adopting a 2-lag VAR specification. In Panel F, we augment our benchmark VAR specifications by introducing market excess returns to control for shocks to risk premia as follows:

$$\tilde{Y}_{i,t} = \begin{bmatrix} r_i^{ex} - r_{US}^{ex} \\ \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix}, \quad (\text{A5})$$

$$Y'_{i,t} = \left[ r_i^{ex} \quad r_{US}^{ex} \quad \sigma_t(\Delta y_i) \quad \sigma_t(\Delta y_{US}) \quad \Delta y_i \quad \Delta y_{US} \quad \sigma_t(\Delta c_i) \quad \sigma_t(\Delta c_{US}) \right]. \quad (\text{A6})$$

As shown in table A1, our main empirical results are quite robust across all of these specifications. For example, the measures of the relative volatility pass-through (second column) are in the 0.50 – 0.65 range, consistent with our benchmark estimates. The size effect on pass-through, i.e., the pass-through being larger when volatility shocks originate in smaller countries, is also a robust finding.

In table A2, we provide further robustness checks for our main results. In Panel A, we show that our VAR results are not specific to the US by substituting the US with a global aggregate. Specifically, we replace US variables with cross-sectional averages of the corresponding variables across G17 countries.

In Panel B, we augment our benchmark VAR specifications by introducing a global

**Table A1: Robustness of Pass-Through Results (I)**

	Relative Vol Shock:	Origin of Vol Shock:	
		US	Foreign
<b>Panel A: Different Cholesky Order</b>			
<i>US/G7</i>	0.52 [0.47; 0.56]	0.49 [0.43; 0.54]	0.56 [0.50; 0.62]
<i>US/Bottom-10 G17</i>	0.61 [0.56; 0.66]	0.50 [0.44; 0.57]	0.73 [0.66; 0.79]
<b>Panel B: Correlated Level and Vol Shocks</b>			
<i>US/G7</i>	0.51 [0.47; 0.55]	0.55 [0.50; 0.60]	0.57 [0.50; 0.63]
<i>US/Bottom-10 G17</i>	0.65 [0.61; 0.68]	0.58 [0.53; 0.64]	0.86 [0.80; 0.92]
<b>Panel C: Controlling for GDP, Consumption, and NX</b>			
<i>US/G7</i>	0.60 [0.55; 0.64]	0.55 [0.50; 0.61]	0.65 [0.60; 0.71]
<i>US/Bottom-10 G17</i>	0.66 [0.62; 0.71]	0.57 [0.51; 0.63]	0.76 [0.70; 0.82]
<b>Panel D: Pooled Vol Estimation</b>			
<i>US/G7</i>	0.50 [0.46; 0.54]	0.47 [0.42; 0.51]	0.57 [0.50; 0.64]
<i>US/Bottom-10 G17</i>	0.61 [0.56; 0.65]	0.51 [0.45; 0.57]	0.72 [0.66; 0.78]
<b>Panel E: VAR(2)</b>			
<i>US/G7</i>	0.59 [0.55; 0.62]	0.50 [0.55; 0.60]	0.63 [0.58; 0.68]
<i>US/Bottom-10 G17</i>	0.65 [0.61; 0.69]	0.57 [0.51; 0.64]	0.74 [0.68; 0.80]
<b>Panel F: Controlling for Risk Premia</b>			
<i>US/G7</i>	0.52 [0.48; 0.56]	0.50 [0.44; 0.55]	0.57 [0.50; 0.63]
<i>US/Bottom-10 G17</i>	0.63 [0.58; 0.68]	0.50 [0.43; 0.56]	0.78 [0.71; 0.84]

Notes: The second column refers to pass-through of relative output volatility shocks estimated from modified versions of the VAR specified in equations (2.2)–(2.3). The rightmost two columns report pass-through with respect to volatility shocks affecting either the US or the remaining countries (based on modified versions of equations (2.6)–(2.7)). In each panel, we report results from a specific robustness check. In Panel A, variables are sorted as in equations (A3)–(A4). In Panel B, we allow for a non-zero correlation between level and vol shocks in the estimation of the system of equations (2.1). In Panel C, we modify the first equation of (2.1) by including lagged output and consumption growth and the net-exports-to-output ratio. In Panel D, we estimate macroeconomic volatility assuming that the volatility parameters are the same across countries other than the US. In Panel E, we estimate our VARs with 2 lags. In Panel F, we add equity returns to our VAR, as specified in equations (A5)–(A6). We report 95% credible intervals in brackets. Our quarterly data range from 1971:q1 to 2013:q4.

volatility component as follows:

$$\tilde{Y}_{i,t} = \begin{bmatrix} \sigma_t(\Delta y_{global}) \\ \sigma_t(\Delta y_i) - \sigma_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix}, \quad (A7)$$

$$Y'_{i,t} = \left[ \sigma_t(\Delta y_{global}) \quad \sigma_t(\Delta y_i) \quad \sigma_t(\Delta y_{US}) \quad \Delta y_i \quad \Delta y_{US} \quad \sigma_t(\Delta c_i) \quad \sigma_t(\Delta c_{US}) \right]. \quad (A8)$$

We consider both an average measure computed across our 17 countries, and a measure based on the first principle component (henceforth ‘‘PC’’) of country volatilities from our cross-section.

Since our VAR specifications impose that the parameters of interest must be the same across countries, this estimation procedure implicitly assumes that all countries have the same exposure to the global volatility component. We relax this assumption by replacing total output volatility with idiosyncratic (i.e., purely country-specific) volatility in equations (2.3) and (2.7), as specified below:

$$\tilde{Y}_{i,t} = \begin{bmatrix} \hat{\sigma}_t(\Delta y_i) - \hat{\sigma}_t(\Delta y_{US}) \\ \Delta y_i - \Delta y_{US} \\ \sigma_t(\Delta c_i) - \sigma_t(\Delta c_{US}) \\ \Delta c_i - \Delta c_{US} \\ \Delta(NX/Y)_i - \Delta(NX/Y)_{US} \end{bmatrix}, \quad (A9)$$

and

$$Y'_{i,t} = \left[ \hat{\sigma}_t(\Delta y_i) \quad \hat{\sigma}_t(\Delta y_{US}) \quad \Delta y_i \quad \Delta y_{US} \quad \sigma_t(\Delta c_i) \quad \sigma_t(\Delta c_{US}), \right] \quad (A10)$$

where the idiosyncratic volatility for country  $i$ ,  $\hat{\sigma}_t(\Delta y_i)$  is the residual of the following regression:

$$\sigma_t(\Delta y_i) = \bar{\sigma}^i + \beta_{\sigma}^i \sigma_t(\Delta y_{global}) + \hat{\sigma}_t(\Delta y_i). \quad (A11)$$

We report our results in Panel C.

All of these specifications confirm our two main findings: (i) among large countries, the pass-through is about 0.50; and (ii) the pass-through is economically and statistically larger for smaller countries.

**Table A2: Robustness of Pass-Through Results (II)**

	Relative Vol Shock:	Origin of Vol Shock:	
		US	Foreign
<b>Panel A: Global Benchmark</b>			
<i>Global Benchmark/G7</i>	0.54 [0.49; 0.58]	0.59 [0.45; 0.72]	0.53 [0.48; 0.58]
<i>Global Benchmark/Bottom-10 G17</i>	0.69 [0.63; 0.75]	0.61 [0.45; 0.77]	0.69 [0.63; 0.75]
<b>Panel B: Controlling for Global Vol</b>			
<i>US/G7 (Average)</i>	0.52 [0.48; 0.57]	0.51 [0.45; 0.57]	0.54 [0.48; 0.61]
<i>US/Bottom-10 G17 (Average)</i>	0.61 [0.57; 0.66]	0.52 [0.45; 0.59]	0.70 [0.63; 0.77]
<i>US/G7 (PC)</i>	0.53 [0.49; 0.57]	0.53 [0.48; 0.59]	0.53 [0.47; 0.60]
<i>US/Bottom-10 G17 (PC)</i>	0.63 [0.59; 0.68]	0.58 [0.52; 0.65]	0.68 [0.61; 0.74]
<b>Panel C: Heterogenous Exposure to Global Vol</b>			
<i>US/G7 (Average)</i>	0.56 [0.51; 0.60]	0.55 [0.50; 0.61]	0.57 [0.50; 0.64]
<i>US/Bottom-10 G17 (Average)</i>	0.66 [0.61; 0.70]	0.59 [0.52; 0.65]	0.73 [0.66; 0.80]
<i>US/G7 (PC)</i>	0.54 [0.49; 0.58]	0.53 [0.47; 0.59]	0.55 [0.48; 0.62]
<i>US/Bottom-10 G17 (PC)</i>	0.63 [0.58; 0.68]	0.59 [0.52; 0.65]	0.67 [0.61; 0.74]

Notes: The second column refers to pass-through measures obtained from variations of the VAR specified in equations (2.2)–(2.3) with respect to a shock to relative output volatility. The rightmost two columns report pass-through measures based on the modified estimates of the VAR in equations (2.6)–(2.7) with respect to volatility shocks affecting either the US or the remaining countries. In Panel A, we substitute US data with data from a global benchmark defined as the average of the corresponding series across all countries. In Panel B, we add a global vol measure to our VAR, as specified in equations (A7)–(A8). We consider both a cross-country average (*Average*) of volatility and a measure obtained through a principal components approach (*PC*). In Panel C, we focus on the VAR specified in equations (A9)–(A10) with country-specific volatility processes estimated as in equations (A11). We report 95% credible intervals in brackets. Our quarterly data range from 1971:q1 to 2013:q4.

## Appendix B. Standard Moments from the Model

In table B1, we focus on unconditional moments typically targeted in the international finance literature. Our benchmark calibration conforms well with our data, both with and without volatility shocks. The adoption of CRRA preferences generates well-known puzzles: (i) the market price of risk is excessively low; (ii) the risk-free rate is too high; and (iii) international trade is modest. In our model the net exports are not as volatile

**Table B1: Standard Unconditional Moments**

	G-17 Data		Model		
	Avg.	Quintiles [ 1 <sup>st</sup> ; 4 <sup>th</sup> ]	Bench- mark	No TVV ( $\sigma_\sigma = 0$ )	CRRA ( $\gamma = 7$ )
$corr(\Delta c, \Delta c^*)$	0.25	[0.13; 0.33]	0.38	0.37	0.74
$\sigma(\Delta c)(\%)$	1.67	[1.34; 2.47]	1.85	1.82	1.64
$\sigma(\Delta c)/\sigma(\Delta y)$	0.88	[0.57; 0.82]	0.93	0.94	0.83
$ACF1(\Delta c)$	0.17	[-0.16; 0.31]	0.06	0.07	0.08
$\sigma(M)/E(M)(\%)$	–	–	47.86	47.85	11.49
$\sigma(\Delta e)(\%)$	10.50	[10.2; 11.4]	12.80	12.65	8.31
$E(r^f)(\%)$	1.35	[1.44; 2.41]	2.17	2.19	14.91
$\sigma(r^f)(\%)$	1.79	[1.61; 2.27]	0.33	0.33	3.47
$corr(r^f, r^{f*})$	0.51	[0.37; 0.56]	0.91	0.92	0.98
$\sigma(\Delta(NX/Y))/\sigma(\Delta y)$	0.70	[0.67; 0.97]	0.32	0.32	0.16

Notes: This table reports key moments for real consumption ( $C$ ), output ( $Y$ ), the exchange rate ( $E$ ), the risk-free rates ( $R^f$ ), the net-export-to-output ratio ( $NX/Y$ ), and the stochastic discount factor ( $M$ ). Small letters refer to log-units; changes are denoted by ‘ $\Delta$ ’; foreign variables are marked by ‘\*’. We denote expectation, standard deviation, correlation, and first order auto-correlation by  $E$ ,  $\sigma$ ,  $corr$ , and  $ACF1$ , respectively. The data refer to G-17 countries and are described in section 2.1. For each country, we compute the moments of interest over the post-Bretton Wood period, 1971:Q1–2013:Q4, as detailed in section 2.1. For each moment, we report (i) its GDP-weighted average across countries; and (ii) its first and fourth cross-country quintiles. The entries from the model are obtained from 100 repetitions of small samples. Our benchmark quarterly calibration is reported in table 4.

as in our G17 dataset, but they are twice as volatile compared to the CRRA case.

## Appendix C. Volatility Frontiers

Given our interest in the volatility pass-through and in the volatility disconnect anomaly, we pay particular attention to the response of the volatility of consumption growth rates,  $\sigma_t(\Delta c_{t+1}^i)$ , to the three sources of risk that are present in the economy. Without loss of generality, we focus on the conditional volatility of the growth rate of consumption of the home country.

The consumption growth in the home country can be expressed in terms of the primitive endowment processes and the share dynamics:

$$\Delta c_{t+1} = \Delta c_{t+1}^{aut} + f(S_{t+1}) - f(S_t), \quad (C1)$$

where  $\Delta c_{t+1}^{aut} := \alpha \Delta X_{t+1} + (1 - \alpha) \Delta Y_{t+1}$  is the consumption growth rate that would prevail under financial autarky, and

$$f(S) := \log \left( \frac{\alpha}{1 - \alpha} \right)^{2\alpha - 1} + \log \left[ \frac{S}{\left(1 + \frac{\alpha}{1 - \alpha} S\right)^\alpha \left(1 + \frac{1 - \alpha}{\alpha} S\right)^{1 - \alpha}} \right]$$

captures the effects of relative size, as measured by  $S_t$ <sup>14</sup>. Note that under financial autarky there is no dynamic redistribution of resources across countries, that is, the reallocation effect is absent. Also, it can be easily shown that  $f' > 0$ ,  $f'' < 0$ ,  $f''' > 0$ , and that  $\lim_{S \rightarrow \infty} f' = 0$ . Equivalently, the  $f$  function is increasing and concave in size,  $S$ , and it is relatively flatter (steeper) for larger (smaller) countries.

A first-order approximation of consumption growth at date  $t + 1$  about date  $t$ 's ratio of pseudo-Pareto weights yields

$$\begin{aligned} \sigma_t^2(\Delta c_{t+1}) &\approx \sigma_t^2(\Delta c_{t+1}^{aut}) + [f'(S_t)]^2 \sigma_t^2(S_{t+1} - S_t), \\ &+ 2 \cdot \text{cov}_t(\Delta c_{t+1}^{aut}, f'(S_t) \cdot (S_{t+1} - S_t)). \end{aligned} \quad (\text{C2})$$

The variance of consumption growth is thus driven by the variation in the fundamental endowment processes, variation in size, and the covariance between the two. To help illustrate the economic role of these channels, we consider two polar cases.

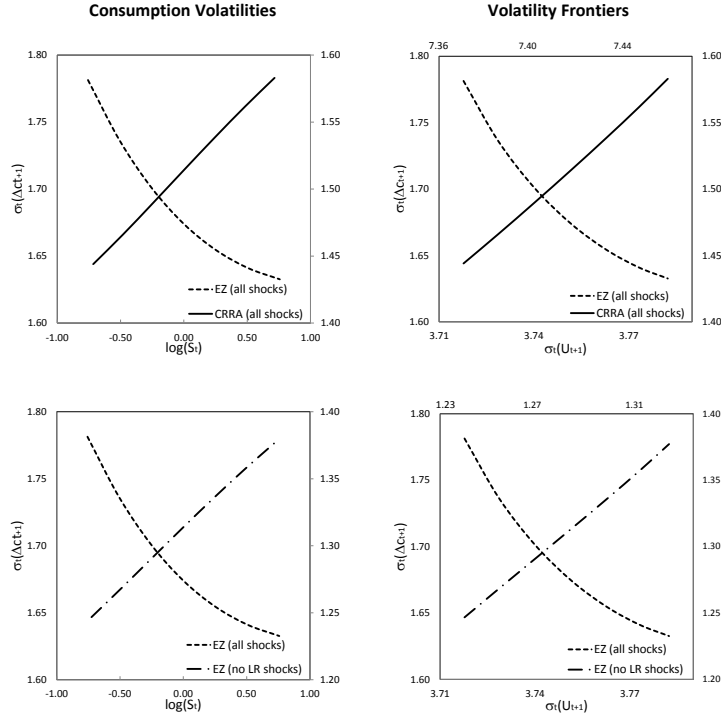
In the first case, assume that either there are no news shocks (i.e.,  $\sigma_\sigma = \sigma_z = 0$ ) or they are not priced (i.e., agents have CRRA preferences). Let the risk aversion coefficient be strictly greater than one, meaning that the risk-sharing motive is strong enough. The covariance term in equation (C2) is negative for  $S \in (0, \infty)$  because the size of the home country increases upon the arrival of a relative negative shock (see equation (3.12)). Furthermore, this negative covariance is greater than  $[f'(S_t)]^2 \cdot \sigma_t^2(S_{t+1} - S_t)$  because the volatility of consumption under complete markets,  $\sigma_t(\Delta c_{t+1})$ , is smaller than the volatility of consumption under portfolio autarky,  $\sigma_t(\Delta c_{t+1}^{aut})$ . This is a common prediction of frictionless risk-sharing models (see, inter alia, Cole and Obstfeld (1991)).

As a result, with respect to only short-run shocks: (i) the reallocation channel *reduces* consumption growth volatility; (ii) this effect is stronger for smaller countries, since  $f'' < 0$ ; and (iii) the consumption volatility frontier is *upward* sloping with respect to country size. These findings are consistent with the model of Hassan (2013), in which small countries feature a lower consumption volatility than large countries.

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<sup>14</sup>This result is obtained from equations (3.1) and (3.11). See Cole and Obstfeld (1991) and Colacito and Croce (2013) for the derivations.





**Figure C1. Variance Frontiers.** The left panels report the conditional volatility of the growth rate of consumption in the home country ( $\sigma_t(\Delta c_{h,t+1})$ ) as a function of the logarithm of the ratio of pseudo-Pareto weights ( $s_t$ ). In the right panels we replace the logarithm of the ratio of pseudo-Pareto weights with the associated conditional volatility of the normalized continuation utility in the home country (i.e.,  $\sigma_t(U_{h,t+1}/(X_t^\alpha Y_t^{1-\alpha}))$ ). In the left panels, the left (right) axis reports the values for our benchmark (alternative) calibration. In the right panels, the values for our benchmark (alternative) calibration are reported on the left and bottom (top and right) axes. Across all cases, we keep both the exogenous long-run components and the exogenous volatility processes fixed at their unconditional mean.

The second extreme case that we consider is the one in which there are only news shocks. By definition, pure news shocks realized at time  $t+1$  do not change the level of  $\Delta c_{t+1}^{aut}$ . As a consequence, the covariance term in equation (C2) is null. If news shocks are priced, they promote an international reallocation of resources at time  $t+1$ , implying that the conditional volatility of  $S_{t+1}$  is strictly larger than zero. Equivalently, the reallocation channel *increases* the conditional volatility of consumption. Since  $f'' < 0$ , the intensity of this channel is stronger for small countries. Thus, the volatility of consumption growth inherits the properties of  $f'(S)$  and the frontier is *downward* sloping in country size.

In our benchmark calibration with recursive preferences, news shocks are of first-order importance for the international reallocation of resources, which explains why our equilibrium consumption volatility frontier is downward sloping (figure C1, top-left panel). As a result, if a country experiences either a positive short-run shock or a positive

long-run shock, its relative size decreases and the volatility of its consumption growth rate rises, whereas the opposite holds for the other country (figure 4(a), second row).

Since the recursive risk-sharing mechanism of this economy is characterized by the agents' willingness to trade off size for a smoother future consumption profile,  $\sigma_t(U_{t+1})$ , savings are dynamically adjusted to achieve long-run consumption smoothing at the cost of increasing short-run consumption volatility,  $\sigma_t(\Delta c_{t+1})$  (figure C1, top-right panel).

Consistent with our analysis of equation (C2), this trade-off is absent when we use CRRA preferences, as news shocks are not priced and hence the consumption volatility frontier is upward sloping (figure C1, top panels, solid lines). This trade-off also disappears when we retain recursive preferences but remove long-run shocks. In this case, short-run level shocks dominate and the consumption volatility frontier is again upward sloping (figure C1, bottom panels), implying that long-run consumption smoothing co-exists with smoother short-run consumption profiles.

A second important insight from figures 4(a) and C1 is that the absolute change in volatility for the country that is affected by a positive level shock (i.e., negative adjustment in its pseudo-Pareto weight) is larger than the absolute change in the volatility for the other country. Equivalently, the short-run consumption volatility frontier is convex, due to the convexity of  $f'$  (recall that  $f''' > 0$ ). As a result, the variation of the share of world consumption produces greater variability in the consumption aggregate of smaller countries, an important feature that allows us to explain our evidence on the volatility pass-through.