Risk-Adjusted Capital Allocation and Misallocation*

Joel M. David† Lukas Schmid‡ David Zeke§

USC Duke USC

August 23, 2018

Abstract

We develop a theory linking "misallocation," i.e., dispersion in static marginal products of capital (MPK), to systematic investment risks. In our setup, firms differ in their exposure to these risks, which we show leads naturally to heterogeneity in firm-level risk premia and, more importantly, MPK. The theory predicts that cross-sectional dispersion in MPK (i) depends on cross-sectional dispersion in risk exposures and (ii) fluctuates with the price of risk, and thus is countercyclical. We empirically evaluate these predictions and document strong support for them. We devise a strategy to quantify variation in firm-level risk exposures using data on the dispersion of expected stock market returns. Our estimates imply that risk considerations explain about 40% of observed MPK dispersion among US firms and in particular, can rationalize a large persistent component in firm-level MPK deviations. Our framework provides a sharp link between cross-sectional asset pricing, aggregate volatility and long-run economic performance – MPK dispersion induced by risk premium effects, although not *prima facie* inefficient, lowers the average level of aggregate TFP by as much as 8%, suggesting large "productivity costs" of business cycles.

*We thank Frederico Belo, Vasco Carvalho, Greg Duffee, Emmanuel Farhi, Francois Gourio, John Haltiwanger, Şebnem Kalemli-Özcan, Matthias Kehrig, Leonid Kogan, Deborah Lucas, Ben Moll, Stijn Van Nieuwerburgh, Stavros Panageas, Dimitris Papanikolaou, Diego Restuccia, John Shea, Venky Venkateswaran, Neng Wang and Amir Yaron for their helpful suggestions, Simcha Barkai, Harjoat Bhamra, Wei Cui, Brent Glover, Ellen McGrattan and Christian Opp for insightful discussions, and many seminar and conference participants for useful comments.

†joeldavi@usc.edu.

‡lukas.schmid@duke.edu.

§zeke@usc.edu.

1 Introduction

A large and growing body of work has documented the "misallocation" of resources across firms, measured by dispersion in the marginal product of inputs into production. Further, the failure of marginal product equalization has been shown to have potentially sizable negative effects on aggregate outcomes, such as productivity and output. Recent studies have found that even after accounting for a host of leading candidates – for example, adjustment costs, financial frictions, or imperfect information – a substantial portion of observed misallocation seems to stem from other firm-specific factors, specifically, of a type that are orthogonal to firm fundamentals and are extremely persistent (if not permanent) to the firm.[1] Identifying exactly what – if any – underlying economic forces can lead to this type of distortion has proven puzzling.

In this paper, we propose, empirically test and quantitatively evaluate just such a mechanism. Our theory links capital misallocation to systematic investment risks. To the best of our knowledge, we are the first to make the connection between standard notions of the risk-return tradeoff faced by investors and the resulting dispersion in the marginal product of capital (MPK) across firms. Indeed, our framework provides a natural way to translate the findings of the rich literature on cross-sectional asset pricing into the implications for the allocation of capital across firms. Further, we are able to quantify the effects of risk considerations – e.g., dispersion in risk premia across firms and the nature of business cycle volatility – on long-run macroeconomic outcomes, such as aggregate total factor productivity (TFP). Through the marginal product dispersion they induce, risk premia effects – although not *prima facie* inefficient – depress the average level of achieved TFP in the economy, leading to a previously unexplored "productivity cost" of business cycles in the spirit of Lucas (1987).[2]

Our point of departure is a standard neoclassical model of firm investment in the face of both aggregate and idiosyncratic uncertainty. Firms discount future payoffs using a stochastic discount factor that is also a function of aggregate conditions. Critically, this setup implies that firms optimally equalize not necessarily MPK, but expected, appropriately discounted, MPK.[3] With little more structure than this, the framework gives rise to an asset pricing equation governing the firm's expected MPK – firms with higher exposure to the aggregate risk factors require a higher risk premium on investments, which translates into a higher expected MPK. This firm-specific risk premium appears exactly as what would otherwise be labeled a persistent "distortion" (in the sense that it shows up as a persistent firm-specific wedge in the Euler

[1]See, e.g., David and Venkateswaran (2017). We discuss the literature in more detail below.

[2]Our analysis is also reminiscent of the approach in Alvarez and Jermann (2004), who use direct data on asset prices to measure the cost of aggregate fluctuations.

[3]Importantly, this is a statement only about expected MPK; realized MPK may differ across firms for additional reasons, i.e., uncertainty over future shocks.

equation). In fact, the model implies a beta pricing equation of exactly the same form that is often used to price the cross-section of stock market returns. The equation simply states that a firm's expected MPK should be linked to the exposure of its MPK to systematic risk (i.e., the firm's "beta"), and the price of that risk.

We begin our analysis by demonstrating that the simple logic of the pricing equation contains substantial empirical content. Specifically, we state and empirically investigate four key predictions of our general framework – (i) exposure to standard risk factors priced in asset markets is an important determinant of expected MPK, (ii) movements in factor risk prices are linked to fluctuations in the conditional expected MPK, (iii) MPK dispersion is positively related to beta dispersion, and (iv) movements in factor risk prices are linked to fluctuations in MPK dispersion. We use a combination of firm-level production and stock market data to provide empirical support for each of these predictions. For example, (i) high MPK firms tend to offer high expected stock returns, suggesting that higher MPK is linked to higher exposure to systematic risk, and further, direct measures of these exposures are positively related to levels of MPK (ii) common return predictors such as credit spreads and the aggregate price/dividend ratio predict fluctuations in mean firm-level MPK, (iii) in the cross-section, industries with higher dispersion in factor exposures, i.e., betas, have higher dispersion in MPK, and (iv) both MPK dispersion and the return on a portfolio of high-minus-low MPK stocks contain predictable, and in fact countercyclical, components, as indicated by the same return predictors as in (ii).

After establishing these empirical results, we interpret them and gauge their magnitudes through the lens of a quantitative model. To that end, we enrich our theory of firm investment dynamics by explicitly linking the sources of uncertainty to idiosyncratic and aggregate productivity risk. We add two key elements to this framework – first, an exogenously specified stochastic discount factor designed to match standard asset pricing moments, as has become standard in the cross-sectional asset pricing literature (e.g., Zhang (2005) and Gomes and Schmid (2010)). Second, we allow for *ex-ante* cross-sectional heterogeneity in exposure, that is, beta, with respect to the systematic productivity risk. In other words, the productivity of high beta firms is highly sensitive to the realization of aggregate productivity, low beta firms have low sensitivity, and indeed, the productivity of firms with negative beta may move countercyclically. The investment side of the model is analytically tractable and yields sharp characterizations of firm investment decisions and MPK.

This setup is consistent with the key empirical results described above, namely, firm-level expected MPKs are dependent on exposures to the aggregate productivity shock (the systemic risk factor in the economy) and due to the countercyclical nature of factor risk prices, are countercyclical, as is the cross-sectional dispersion in expected MPK. Further, we derive an expression for aggregate TFP, which is a strictly decreasing function of MPK dispersion. By inducing persistent MPK dispersion, either greater cross-sectional variation in factor risk exposures and/or a higher price of risk reduce the average (long-run) level of achieved TFP. Thus, our model provides a novel, directly quantifiable link between financial market conditions, i.e., the nature of aggregate risk, and longer-run economic performance.

In our framework, the strength of these connections rely on three key parameters – the degree of heterogeneity in firm-level risk exposures and the magnitude and time-series variation in the price of risk. We devise a novel empirical strategy to pin down these parameters using salient moments from firm-level and aggregate stock market data, specifically, (i) the cross-sectional dispersion in expected stock returns, (ii) the average market equity premium and (iii) the average market Sharpe ratio. We use a linearized version of our model to derive sharp analytical expressions for these moments and show that they are tightly linked to the structural parameters. The latter two pin down the long-run level and volatility of the price of risk and the first identifies the cross-sectional dispersion in firm-level risk exposures. Indeed, in some simple cases of our model, the dispersion in expected MPK induced by risk premium effects is directly proportional to the dispersion in expected stock returns – intuitively, both of these moments are determined by cross-sectional variation in betas.

Before quantitatively evaluating this mechanism, we explore the effects of adding other investment frictions to our environment, specifically, capital adjustment costs. Although they do not change the main insights from our simpler model, we uncover an important interaction between these costs and risk premia – namely, adjustment costs actually amplify the effects of beta variation on MPK dispersion. Intuitively, beta dispersion leads to persistent differences in firm-level capital choices, even if these firms have the same average level of productivity. Adjustment costs further increase the dispersion in capital, which leads to even larger effects on MPK. On their own, adjustment costs do not lead to any persistent dispersion in firm-level MPK, but they augment the effects of other factors that do, such as the variation in risk premia we analyze here.

We apply our methodology to data on US firms from Compustat/CRSP and macro/financial aggregates, e.g., productivity and stock market returns. Our estimates reveal substantial variation in firm-level betas and a sizable price of risk – together, these imply a significant amount of risk-induced MPK dispersion. For example, if this were the only source of MPK dispersion, variation in risk premia would account for about 15% of total dispersion among Compustat firms. In the presence of adjustment costs, the figure is notably higher – in this case, risk premia effects explain 44% of total dispersion in MPK. Importantly, the dispersion from this channel is persistent – in other words, risk effects manifest themselves as persistent MPK deviations at the firm level, exactly of the type that have been shown to compose a large portion of observed misallocation. Indeed, our results can account for as much as 67% of this permanent component in the data. The consequences of these values for the long-run level of aggregate TFP are significant – cross-sectional variation in risk reduces TFP by as much as 8%. Note that this represents a quantitative estimate of the impact of the rich set of findings in the cross-sectional asset pricing literature on macroeconomic performance and further, a new connection between the nature of business cycle volatility and long-run outcomes in the spirit of Lucas (1987). Here, higher aggregate volatility leads to greater aggregate risk, increasing dispersion in required rates of return and MPK and thus reducing TFP. Our findings suggest these "productivity costs" of business cycles may be substantial.

Our estimates also imply a significant predictable countercyclical element in expected MPK dispersion. For example, our parameterized model produces a correlation between the cross-sectional variance in expected MPK and the state of the business cycle (measured by the aggregate productivity shock) of -0.25. To put this number in context, the correlation between MPK dispersion and aggregate productivity in the data is about -0.27. This result provides a risk-based explanation for the otherwise puzzling observation, made forcefully by Eisfeldt and Rampini (2006), that capital reallocation is procyclical, in spite of the apparently countercyclical potential gains – due to the countercyclical nature of factor risk prices and high beta of high MPK firms, such reallocation in downturns would require capital to flow to the riskiest of firms in the riskiest of times.

Before concluding, we perform three important additional exercises. First, we add to the environment a flexible class of firm-specific "distortions" of the type that have been emphasized in the misallocation literature. These distortions can be correlated or uncorrelated with the idiosyncratic component of firm-productivity and can be fixed or time-varying. To a first-order approximation, we show these additional factors do not affect our results or identification approach. In other words, although observed misallocation may stem from a variety of sources, our empirical strategy to measuring risk premium effects yields an accurate estimate of the contribution of this one source alone.[4] Second, we demonstrate the crucial role of *ex-ante* dispersion in risk exposures in generating a quantitatively realistic dispersion in expected returns. To do so, we examine a model with no beta dispersion, but adjustment costs and potential heterogeneity in other firm-level parameters, for example, curvature of the production function. We find that adjustment costs alone do not lead to significant expected return dispersion. Further, although heterogeneity in firm-level production parameters can generate non-negligible expected return dispersion, it is still only a relatively small fraction of the wide dispersion observed in the data, suggesting that variation in betas is a key ingredient in matching this moment. Third, we provide further, direct evidence on the extent of beta dispersion. Rather than relying on stock

[4] We also analyze distortions that can be correlated with the aggregate shock and show that under plausible assumptions, our approach yields a conservative estimate of risk premium effects.

market data, we compute firm-level betas using production-side data by estimating time-series regressions of measures of firm-level productivity on measures of aggregate productivity. The beta is the coefficient from this regression. We show that this exercise yields beta dispersion on par with the dispersion implied by the cross-section of stock market returns.

Related Literature. Our paper relates to several branches of the literature. First is the large body of work investigating and quantifying the effects of resource misallocation across firms, seminal examples of which include Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). A number of papers have explored the role of particular economic forces in leading to misallocation. For example, Asker et al. (2014) study the role of capital adjustment costs, Midrigan and Xu (2014), Moll (2014) and Buera et al. (2011) financial frictions, Peters (2016) markup dispersion and David et al. (2016) information frictions.[5] Gopinath et al. (2017) and Kehrig and Vincent (2017) show that the interactions of adjustment costs and financial frictions are important in determining the recent dynamics of misallocation in Spain and the extent of misallocation across plants within firms, respectively. David and Venkateswaran (2017) provide an empirical methodology to disentangle various sources of capital misallocation and establish a large role for other firm-specific factors, in particular, ones that are essentially permanent to the firm. We build on this literature by exploring the implications of a different dimension of financial markets for marginal product dispersion, namely, the risk-return tradeoff faced by risk-averse investors. Importantly, our theory generates what appears to be a permanent firm-specific "wedge" exactly of the type found by David and Venkateswaran (2017), but which in our framework is a function of each firm's exposure to aggregate risk. The addition of aggregate risk is a key innovation of our analysis – existing work has typically abstracted from this channel.[6] We show that the link between aggregate risk and observed misallocation is quite tight in the presence of heterogeneous exposures to that risk.

Kehrig (2015) documents in detail the countercyclical nature of productivity dispersion. We build on this finding by relating fluctuations in MPK dispersion to time-series variation in the price of risk. A growing literature, starting with Eisfeldt and Rampini (2006), investigates the reasons underlying the observation that capital reallocation is procyclical. This indeed seems puzzling since, given higher cross-sectional dispersion in MPK in downturns, one should expect to see capital flowing to highly productive, high MPK firms in recessions. Our results bear on that observation by noting that the countercyclical nature of risk prices, in conjunction with

[5] Many papers study the role of firm-specific distortions, e.g., Bartelsman et al. (2013). Restuccia and Rogerson (2017) and Hopenhayn (2014) provide recent overviews of this line of work.

[6] Two important exceptions are Gopinath et al. (2017), who analyze the transitional effects of an interest rate shock on misallocation, and Kehrig (2015), who constructs a model of misallocation over the business cycle featuring overhead inputs. Neither of these papers examines risk premium effects, either because there is no aggregate uncertainty or firms are risk-neutral.

heterogeneity in factor risk exposures, go some way toward potentially reconciling this puzzle.

In a related effort, Binsbergen and Opp (2017) also investigate the implications of asset market considerations for the real economic decisions of firms. They propose a framework in which distortions in agents' subjective beliefs map to "alphas," i.e., cross-sectional mispricings, and through this channel to real efficiency losses. Our analyses are complementary in that where they focus on the implications of mispricing of financial assets for corporate decisions, we investigate the extent of efficient dispersion in firm-level marginal products induced by variation in exposures to aggregate risk factors. Our empirical work establishes a connection between observed marginal products and asset market outcomes and our quantitative work uses a workhorse macroeconomic model of firm dynamics augmented to feature risk-sensitive agents and aggregate risk to evaluate the implications of this insight. One of our key messages shares a common theme with the findings in Binsbergen and Opp (2017) – financial market considerations can have sizable effects on real outcomes by affecting capital allocation decisions.[7]

Our work exploits the insight, due to Cochrane (1991) and Restoy and Rockinger (1994), that stock returns and investment returns are closely linked. Indeed, under the assumption of constant returns to scale, stock and investment returns effectively coincide. Crucially, for our purposes, investment returns are intimately linked with the marginal product of capital. Balvers et al. (2015) explore and confirm the close albeit more complicated relationship under deviations from constant returns to scale. In this context, our work is closely related to the growing literature that examines the cross-section of stock returns by viewing them from the perspective of investment returns, e.g., Zhang (2005), Gomes et al. (2006) and Liu et al. (2009), and recently forcefully summarized in Zhang (2017). This literature interprets common risk factors such as the Fama and French (1992) factors through firms' investment policies and shows that investment-based factors are priced in the cross-section of returns. Our objective is quite different and in some sense turns that logic on its head, in that we examine investment returns and the marginal product of capital as a manifestation of exposure to systematic risk, most readily measured through stock returns.

2 Motivation

In this section, we lay out a simple version of the standard, frictionless neoclassical theory of investment to motivate our empirical explorations. Section 4 enriches this environment for purposes of our quantitative work.

Firms produce output using capital and labor according to a standard Cobb-Douglas pro-

[7]Relatedly, David et al. (2014) find that risk considerations play an important role in determining the allocation of capital across countries, i.e., can explain some portion of the "Lucas Paradox."

duction function. Labor is chosen period-by-period in a spot market at a competitive wage. At the end of each period, firms choose investment in new capital, which becomes available for production in the following period so that $K_{it+1} = I_{it} + (1-\delta) K_{it}$, where δ is the rate of depreciation. Let $\Pi_{it} = \Pi_{it}(X_t, Z_{it}, K_{it})$ denote the operating profits of the firm – revenues net of labor costs – where X_t and Z_{it} denote aggregate and idiosyncratic shocks to firm profitability, respectively, and K_{it} the firm's level of capital. The analysis can accommodate a number of interpretations of the fundamental shocks, for example, as productivity or demand shifters. Given the Cobb-Douglas technology, the profit function takes a Cobb-Douglas form, is homogeneous in K of degree $\theta \leq 1$ and is proportional to revenues.[8] The marginal product of capital is equal to $MPK_{it} = \theta \frac{\Pi_{it}}{K_{it}}$. The payout of the firm in period t is equal to $D_{it} = \Pi_{it} - I_{it}$.

Firms discount future cash flows using a stochastic discount factor (SDF), M_{t+1}, which may be correlated with the aggregate component of firm fundamentals, i.e., with X_t. We can write the firm's problem in recursive form as

$$V(X_t, Z_{it}, K_{it}) = \max_{K_{it+1}} \Pi_{it}(X_t, Z_{it}, K_{it}) - K_{it+1} + (1-\delta) K_{it} + \mathbb{E}_t \left[M_{t+1} V(X_{t+1}, Z_{it+1}, K_{it+1}) \right] , \tag{1}$$

where $\mathbb{E}_t[\cdot]$ denotes the firm's expectations conditional on time t information. Standard techniques give the Euler equation

$$1 = \mathbb{E}_t \left[M_{t+1} (MPK_{it+1} + 1 - \delta) \right] \quad \forall\, i, t \, . \tag{2}$$

MPK dispersion. An immediate consequence of expression (2) is that MPK (or even expected MPK) need not be equated across firms; rather, it is only appropriately discounted expected MPK that is equalized. To the extent that firms load differently on the SDF, their expected MPK will differ. Assuming a single source of aggregate risk for the sake of illustration, Appendix C derives the following factor model for expected MPK:[9]

$$\mathbb{E}_t[MPK_{it+1}] = \alpha_t + \beta_{it} \lambda_t \, . \tag{3}$$

Here, α_t is the "risk-free" MPK, which equals the riskless user cost of capital, $r_{ft} + \delta$, where r_{ft} is the net risk-free rate, $\beta_{it} \equiv -\frac{\mathrm{cov}_t(M_{t+1}, MPK_{it+1})}{\mathrm{var}_t(M_{t+1})}$ measures the exposure, or loading, of the firm's MPK on the SDF, i.e., the riskiness of the firm, and $\lambda_t \equiv \frac{\mathrm{var}_t(M_{t+1})}{\mathbb{E}_t[M_{t+1}]}$ is the market price of

[8] This structure follows, for example, if firms are perfectly competitive and the production function features decreasing returns to scale or firms are monopolistically competitive and face CES demand curves. We clarify these assumptions in our more detailed model in Section 4

[9] It is straighforward to generalize the results to environments featuring multiple aggregate risk factors, such as the Fama and French (1992) 3-factor model or the Q-factor model of Hou et al. (2015) and Zhang (2017). We provide a multi-factor extension of our baseline theory in Section 6.1.

that risk. In the language of asset pricing, the Euler equation gives rise to a conditional one-factor model for expected MPK. Expression (3) highlights that expected MPK is not necessarily common across firms and is a function of the risk-free rate of return, the firm's beta on the SDF, which may vary across firms, and the market price of risk. The cross-sectional variance of date t conditional expected MPK is then equal to

$$\sigma^2_{\mathbb{E}_t[MPK_{it+1}]} = \sigma^2_{\beta_t} \lambda_t^2 \ , \tag{4}$$

where $\sigma^2_{\beta_t}$ is the cross-sectional variance of conditional betas. The extent to which risk considerations lead to dispersion in expected MPK depends on (i) the cross-sectional variation in firm-level risk exposures and (ii) the price of risk. Further, given persistence in firm-level risk exposures, i.e., beta, the theory can clearly generate persistent differences in firm-level MPK, which are driven by the dispersion in required rates of return across firms.[10]

The strength of the mechanism linking dispersion in MPK to exposure to aggregate risk can be understood by inspection of expression (4) – predicted MPK dispersion is increasing in the dispersion in betas and also in the price of risk, λ. A key observation underlying our analysis is that asset pricing data suggest that risk prices are rather high. For example, a lower bound is given by the Sharpe ratio on the market portfolio, estimated to be around 0.5. However, even easily implementable trading strategies such as those based on value-growth portfolios, or momentum, suggest numbers closer to 0.8, while hedge fund strategies report Sharpe ratios in excess of one. Taken at face value, these numbers suggest the possibility for substantial MPK dispersion – even in frictionless environments – after taking risk exposure in account.

Empirical Predictions. Even under the simple structure we have outlined thus far, the theory has a good deal of empirical content. Specifically, the expressions laid out above contain a number of both cross-sectional and time-series predictions:

1. Exposure to standard risk factors is a determinant of expected MPK. Expression (3) shows that the same factors that determine the cross-section of asset returns – namely, exposure to the SDF – determine the cross-section of MPK. Firms with a higher loading on the SDF, i.e., higher beta, should have higher conditional expected MPK.

[10]To see this more clearly, we can take the unconditional expectation of equation (3) to obtain an approximate expression for the variance of mean MPK as $\sigma^2_{\mathbb{E}[MPK]} \approx \sigma^2_{\beta} \lambda^2$, where $\sigma^2_{\beta} \equiv \sigma^2_{\mathbb{E}[\beta_{it}]}$ denotes the variance of the unconditional MPK factor betas and $\lambda \equiv \mathbb{E}[\lambda_t]$ the unconditional expectation of factor risk prices. The approximation is valid as long as $\text{cov}(\beta_i, \text{cov}(\beta_{it}, \lambda_t))$ is small. In line with the results in Lewellen and Nagel (2006), we find the time-series variation in betas to be quite modest. In the case of constant betas, for example (which we assume in our quantitative model), or if time variation in beta is orthogonal to variation in λ, the expression is exact.

2. Variation in the price of risk, λ_t, leads to predictable variation in mean expected MPK. In particular, the mean conditional expected MPK should increase with the price of risk. This is the time-series implication of expression (3) – holding fixed the distribution of beta, movements in λ_t should positively affect the mean expected MPK. Since the price of risk is known to be countercyclical, this adds a countercyclical element to the mean expected MPK.

3. MPK dispersion is related to beta dispersion. Expression (4) shows that unconditional variation in the cross-section of MPK is proportional to the variation in beta. Segments of the economy, for example, industries, with higher dispersion in beta should display higher dispersion in MPK.

4. MPK dispersion is positively correlated with the price of risk. Expression (4) has a time-series prediction linking MPK dispersion to time variation in the price of risk. For a given degree of cross-sectional dispersion in beta, when required compensation for bearing risk increases, MPK dispersion should increase as well.

Illustrative examples. Section 3 investigates each of these predictions in detail. Before doing so, however, it is useful to consider a number of more concrete illustrative examples (derivations for this section are in Appendix C).

Example 1: no aggregate risk (or risk neutrality). In the case of no aggregate risk, we have $\beta_{it} = 0 \ \forall \ i, t$, i.e., all shocks are idiosyncratic to the firm. Expressions (3) and (4) show that there will be no dispersion in expected MPK and for each firm, $\mathbb{E}_t \left[MPK_{it+1}\right] = r_f + \delta$, which is simply the riskless user cost of capital (which is constant in the absence of aggregate shocks). This is the standard result from the stationary models widely used in the misallocation literature where without additional frictions, expected MPK should be equalized across firms.[11] It is straightforward to show this expression also holds in an environment with aggregate shocks but risk neutral preferences, which implies M_{t+1} is simply a constant (equal to the time discount factor).

Example 2: CAPM. In the CAPM, the SDF is linearly related to the market return, i.e., $M_{t+1} = a - bR_{mt+1}$ for some constants a and b. Because the market portfolio is itself an asset

[11] With time-to build for capital and uncertainty over upcoming shocks there may still be dispersion in *realized* MPK, but not in *expected* terms, and so these forces do not lead to persistent deviations from MPK equalization for a particular firm.

with $\beta = 1$, it is straightforward to derive

$$\mathbb{E}_t \left[MPK_{it+1} \right] = \alpha_t + \underbrace{\frac{\text{cov}_t \left(R_{mt+1}, MPK_{it+1} \right)}{\text{var}_t \left(R_{mt+1} \right)}}_{\beta_{it}} \underbrace{\mathbb{E}_t \left[R_{mt+1} - R_{ft} \right]}_{\lambda_t},$$

i.e., expected MPK is determined by the covariance of the firm's MPK with the market return, which is the the risk factor in this environment. The price of risk is equal to the expected excess return on the market portfolio, i.e., the equity premium (R_{ft} is the risk-free rate of return from period t to $t+1$).

Example 3: CCAPM. In the case that the utility function is CRRA with coefficient of relative risk aversion γ, standard approximation techniques give the pricing equation from the consumption capital asset pricing model:

$$\mathbb{E}_t \left[MPK_{it+1} \right] = \alpha_t + \underbrace{\frac{\text{cov}_t \left(\Delta c_{t+1}, MPK_{it+1} \right)}{\text{var}_t \left(\Delta c_{t+1} \right)}}_{\beta_{it}} \underbrace{\gamma \text{var}_t \left(\Delta c_{t+1} \right)}_{\lambda_t},$$

where Δc_{t+1} denotes log consumption growth. Expected MPK is determined by the covariance of the firm's MPK with consumption growth, which is now the risk factor. The price of risk is the product of the coefficient of relative risk aversion and the conditional volatility of consumption growth.

3 Empirical Results

In this section, we investigate the empirical predictions outlined in Section 2.

Data. Our data come primarily from the Center for Research in Security Prices (CRSP) and Compustat. We use data on nonfinancial firms with common equities listed on the NYSE, NASDAQ, or AMEX over the period 1965 to 2015. We supplement this panel with time-series data on market factors and aggregate conditions related to the price of risk. We use data on the Fama and French (1992) (Fama-French) factors, aggregate dividends and stock market values from Shiller (2005) and two measures of credit spreads: the Gilchrist and Zakrajsek (2012) (GZ) credit spread and the excess bond premium.[12] We measure firm capital stock, K_{it}, as the

[12] Data on the Fama-French factors are from Kenneth French's website, `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`. We obtain updated data on the price/dividend ratio from Robert J. Shiller's website, `http://www.econ.yale.edu/~shiller/` and updated measures of the GZ spread and excess bond premium from Simon Gilchrist's website, `http://people.bu.edu/sgilchri/`.

(net of depreciation) value of property, plant and equipment (Compustat series PPENT) and firm revenue, Y_{it}, as reported sales (series SALE).[13] Ignoring constant terms, which will play no role in our analysis, we measure the marginal product of capital (in logs, henceforth denoted with lowercase) as $mpk_{it} = y_{it} - k_{it}$.[14] Appendix A provides further details on how we construct our dataset and the series that we use.

We can now revisit the main predictions from Section 2.

1. Exposure to standard risk factors is a determinant of expected MPK. We investigate this key implication of our framework in several ways.

Portfolio sorts. First, we examine the relationship between MPK and stock market returns across firms. To focus on the link between MPK and returns, we form MPK-sorted portfolios of firms. This approach follows widespread practice in empirical finance, which has generally moved from addressing variation in individual firm returns to returns on portfolios of firms, sorted by factors that are known to predict returns. In our setting, this procedure proves useful to eliminate firm-specific factors unrelated to MPK that may affect returns and so allows us to hone in on the predictability of excess returns by MPK.[15] We sort firms into five portfolios based on their year t MPK, where portfolio 1 contains low MPK firms and portfolio 5 high MPK ones. The portfolios are rebalanced annually. We then compute the contemporaneous and one-period ahead equal-weighted excess stock return to each portfolio, denoted r_t^e and r_{t+1}^e, respectively.[16] We additionally compute excess returns on a high-minus-low MPK portfolio (MPK-HML), which is an annually rebalanced portfolio that is long on stocks in the highest MPK portfolio and short on stocks in the lowest.

We report the results in Panel A of Table 1. The table reveals a strong relationship between MPK and stock returns – portfolios with higher MPK tend to earn higher excess returns. The first row shows that the difference in contemporaneous returns between high and low MPK firms, i.e., the excess return on the MPK-HML portfolio, is over 8% annually. The second row confirms that this finding does not simply result from the simultaneous response of stock returns and MPK to the realization of unexpected shocks – one-period ahead excess returns are in fact predictable by MPK. Indeed, the predictable spread on the MPK-HML portfolio is almost 5% annually. Both the contemporaneous and future MPK-HML spreads are statistically

[13] Using book assets, a broader notion of firm capital, yields similar results.

[14] Recall that in our setup, operating profits are proportional to revenues, making this a valid measure of the mpk.

[15] Additionally, the portfolio approach helps to eliminate the effects of potential measurement error, for example, in firm-level capital stocks.

[16] When computing future returns, we follow Fama and French (1992) and associate the MPK for fiscal year t with returns from July of year $t + 1$ to June of year $t + 2$.

different from zero at the 99% level. Thus, high MPK tend to offer high stock returns, both in a realized and an expected sense, suggesting that MPK differences reflect exposure to risk factors for which investors demand compensation in the form of a higher rate of return.

Table 1: Excess Returns on MPK-Sorted Portfolios

Portfolio	Low	1	2	3	4	MPK-HML
			Panel A: Not Industry-Adjusted			
r^e_t	7.00**	9.08**	10.67***	12.00***	15.25***	8.25***
	(2.01)	(2.53)	(2.93)	(3.09)	(3.71)	(4.54)
r^e_{t+1}	8.60**	12.27***	13.48***	13.73***	13.48***	4.87***
	(2.48)	(3.47)	(3.80)	(3.62)	(3.36)	(2.81)
			Panel B: Industry-Adjusted			
r^e_t	6.98	8.91**	10.59***	12.28***	15.78***	8.80***
	(1.63)	(2.52)	(3.05)	(3.30)	(3.73)	(9.54)
r^e_{t+1}	11.10***	11.55***	12.71***	12.70***	13.69***	2.59***
	(2.61)	(3.35)	(3.75)	(3.50)	(3.36)	(2.98)

Notes: This table reports stock market returns for portfolios sorted by *mpk*. r^e_t denotes equal-weighted contemporaneous annualized monthly excess stock returns (over the risk-free rate) measured in the year of the portfolio formation from January to December of year t. r^e_{t+1} denotes the analogous future returns, measured from July of year $t+1$ to June of year $t+2$. Industry adjustment is done by de-meaning *mpk* by industry-year and sorting portfolios on de-meaned *mpk*, where industries are defined at the 4-digit SIC code level. t-statistics in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

The focus in the misallocation literature is generally on within-industry variation in the MPK.[17] We therefore also perform industry-adjusted portfolio sorts and find that the relationship between MPK and stock returns is also present within individual industries. To control for industry effects, we demean firm-level *mpk* by subtracting the mean *mpk* within each industry-year, and sort firms based on this de-meaned measure.[18] Panel B of Table 1 reports the within-industry results. The relationship between MPK and stock returns remains strong even when comparing across firms within a particular industry, both in an economic and statistical sense – the MPK-HML contemporaneous excess return is over 8% annually and the future excess return is over over 2.5%. Both are statistically significant at the 99% level.

In Appendix D, we explore a number of variants of Table 1. For example, we expand the number of portfolios, examine measures of unlevered returns and consider longer-horizon future returns. The relationship between MPK and stock returns continues to hold under all these alternatives. We perform double-sorts on size and book-to-market and verify that the return spreads based on MPK are not fully explained by the latter two factors (although they are both

[17]There may be heterogeneity across industries on a number of dimensions, for example, in production function coefficients or industry-level exposure to aggregate shocks.

[18]This is equivalent to extracting industry-year fixed-effects. We define an industry as a 4-digit SIC code and examine industry-year pairs with at least 10 observations.

correlated with MPK). We also present summary statistics of the portfolios across a number of characteristics and consider several additional measurement issues (for example, we show that the results are unlikely to be driven by unmeasured intangible capital).

Measures of risk exposures and expected MPK. The second way we verify the implications of prediction 1 is to directly relate firm MPK to measures of risk exposures. To do so, we estimate regressions of the form

$$mpk_{it+1} = \psi_0 + \psi_\beta \beta_{it} + \zeta_{it+1} \,, \tag{5}$$

where β_{it} is a measure of firm i exposure to aggregate risk at time t. The specification tests whether observable measures of firm-level risk exposures are indeed correlated with higher MPK. We estimate (5) at an annual frequency and lag the right-hand side variables to control for the simultaneous effect of unexpected shocks on contemporaneous measures of beta and MPK. We compute four different measures of these exposures. First, we compute standard CAPM and Fama-French stock market betas (β_{CAPM} and β_{FF}, respectively) by estimating firm-by-firm time-series regressions of firm-level stock returns on the risk factors from each of these models. In the CAPM, the single risk factor is the aggregate market return. The three Fama-French factors are the market return, the return on a portfolio that is long in small firms and short in large ones (SMB), which captures the size premium and the return on a portfolio that is long in high book-to-market firms and short in low ones (HML), which captures the value premium. In each model, the coefficient on the risk factor(s) yields a measure of beta. To obtain a single measure of risk exposure in the multi-factor Fama-French model, we combine the resulting betas into a single value using estimated prices of risk from Fama and MacBeth (1973) regressions. We provide details of these calculations in Appendix A.

With these measures in hand, we are in a position to estimate equation (5). We report the results in columns (1)-(2) in Table 2. Both measures have significant explanatory power for subsequent MPK. For example, the estimate in column (1) implies that each unit increase in beta is associated with a 20% increase in expected MPK.

Both of these measures compute firm-level risk exposures based only on stock market data. Although our theory implies these should be related to MPK (and they have a rich tradition in asset pricing), expression (3) suggests that we look directly at the exposure of firm-level MPK on aggregate risk factors. To do so, we perform the same two exercises just described, but instead using firm-level MPK – namely, we regress *mpk* on the market return and the three Fama-French factors to obtain two direct measures of MPK exposure to aggregate risk ($\beta_{CAPM,MPK}$ and $\beta_{FF,MPK}$) and estimate specification (5) using these measures as the predictive

Table 2: Predictive Regressions of MPK on Aggregate Risk Exposures

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_{CAPM}	0.209***				0.014***			
	(51.53)				(3.48)			
β_{FF}		0.068***				0.005***		
		(63.14)				(4.91)		
$\beta_{CAPM,MPK}$			0.065***				0.024***	
			(13.77)				(4.31)	
$\beta_{FF,MPK}$				4.005***				1.097***
				(25.00)				(6.16)
Observations	108571	107845	81559	81062	103488	102820	76832	76351
R^2	0.024	0.036	0.002	0.008	0.059	0.059	0.065	0.065
Firms	10270	10229	8687	8655	9991	9941	8406	8380
F.E.	No	No	No	No	Yes	Yes	Yes	Yes
Controls	No	No	No	No	Yes	Yes	Yes	Yes

Notes: This table reports the results of a panel regression of year-ahead *mpk* regressed on measures of firm exposure to aggregate risk. Each observation is a firm-year. F.E. denotes the presence of industry-year fixed effects. When we include fixed-effects, we cluster standard errors by industry-year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

variable. We report the results in columns (3) and (4) of Table 2. The table shows that, similar to stock market betas, firm-level "MPK betas" are significant predictors of future firm MPK. In sum, our findings in Table 2 confirm the key implications of expression (3): firm-level risk exposures – measured using stock market or MPK exposures – are significant determinants of firm-level expected MPK.

In columns (5)-(8) of Table 2, we estimate analogous regressions with the addition of industry-year fixed-effects and a set of standard firm-level controls, namely, market capitalization, book-to-market ratio, profitability, and market leverage.[19] All of the beta coefficients remain positive and statistically significant.

2. Variation in the price of risk, λ_t, leads to predictable variation in mean expected MPK. Expression (3) implies that the price of risk should positively predict the level of expected MPK. To test this implication, we estimate time-series regressions of the form:

$$\mathbb{E}\left[mpk_{it+1}\right] = \psi_0 + \psi_1 x_t + \zeta_{t+1} \ , \tag{6}$$

where $\mathbb{E}\left[mpk_{it+1}\right]$ denotes the average mpk in period $t+1$ and x_t denotes three different

[19] We describe these series in Appendix A.

measures related to the price of risk: the price/dividend ratio (PD) on the aggregate stock market, and two measures of credit spreads – the Gilchrist and Zakrajsek (2012) (GZ) spread, a high-information and duration-adjusted measure of the mean credit spread and the excess bond (EB) premium, which measures the portion of the GZ spread not attributable to default risk.[20] These are standard proxies for risk prices that have been widely used in the literature. We estimate specification (6) using quarterly data on these measures, where the left-hand side variable is one year (four-quarter) ahead *mpk*.[21] Table 3 reports the results of these regressions. In line with the theory, column (1) shows that the PD ratio (likely negatively correlated with the price of risk) predicts lower future MPK, while columns (2) and (3) show that the GZ spread and the EB premium (likely positively correlated with the price of risk) predict higher future MPK. Thus, the table confirms that time-variation in risk premia tend to forecast future levels of MPK.

Table 3: Predictability of Mean MPK

	(1)	(2)	(3)
PD Ratio	-0.341***		
	(-3.24)		
GZ Spread		4.457***	
		(3.44)	
EB Premium			7.041***
			(3.20)
Observations	166	166	166
R^2	0.120	0.122	0.100

Notes: This table reports time-series regressions of four-quarter ahead average *mpk* on measures of the price of risk. t-statistics are in parentheses, which are computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

3. MPK dispersion is related to beta dispersion. Expression (4) implies that across groups of firms or segments of the economy, dispersion in expected MPK should be positively related to dispersion in risk exposures. We investigate this prediction using variation in the dispersion of firm-level betas and expected stock market returns across industries. Specifically, for each industry in each year, we compute the standard deviation of MPK, $\sigma(mpk)$, expected stock returns, $\sigma(\mathbb{E}[r])$, and beta, $\sigma(\beta)$. We then estimate a pooled regression of industry-level MPK dispersion on the dispersion in expected returns and betas, i.e., we perform regressions of the

[20]We extract the cyclical component of the PD ratio and mean *mpk* using a one-sided Hodrick-Prescott filter. The credit spread measures do not exhibit significant longer-term trends.

[21]To control for the changing composition of firms, for each quarter, we include only firms that were present in the previous quarter and calculate changes in the mean *mpk* for these firms. We then use those changes to construct a composition-adjusted series for mean *mpk* which is unaffected by new additions or deletions from the dataset. We further detail this procedure in Appendix A.

form:

$$\sigma\left(mpk_{jt+1}\right) = \psi_0 + \psi_1 \sigma\left(x_{jt}\right) + \zeta_{jt+1} \quad x_{jt} = \mathbb{E}\left[r_{jt}\right], \beta_{jt} ,$$

where j denotes industry. Again, to avoid potential simultaneity biases from the realization of shocks, we lag the independent variables (dispersion in expected returns and betas) by a year.

Table 4 reports the results of these regressions and demonstrates that indeed, industries with higher dispersion in expected stock returns and beta exhibit greater dispersion in MPK. Column (1) reveals this fact using expected returns calculated from the Fama-French 3 factor model.[22] Variation in expected return dispersion predicted by the Fama-French model explains over 20% of the variation in MPK dispersion across industry-year cells. Column (2) estimates a regression of MPK dispersion on dispersion in each of the three individual factors – variation in the beta on each factor is significantly related to MPK dispersion. Next, we repeat the exercise using dispersion in our measures of MPK betas (described above) as the right-hand side variables. The results in column (3) show that industries with greater dispersion in MPK betas (on each of the Fama-French factors) exhibit greater dispersion in MPK. Columns (4)-(6) perform analogous regressions with the addition of year fixed-effects and a number of controls capturing additional measures of firm heterogeneity within industries – the standard deviations of profitability, size, book-to-market, and market leverage. Across these specifications, measures of within-industry heterogeneity in expected returns and aggregate risk exposures remain positive and significant predictors of within-industry dispersion in MPK.[23]

4. MPK dispersion is positively correlated with the price of risk. Expression (4) implies that the price of risk is positively related to MPK dispersion. We investigate this prediction in two ways. First, we show that the indicators of the price of risk considered before (the PD ratio, GZ spread, and EB premium) predict time-series variation in MPK dispersion. Second, we show that the expected return on the high-minus-low MPK portfolio is also predicted by these measures of the price of risk.

To perform these tests, we estimate regressions of the form

$$y_{t+1} = \psi_0 + \psi_1 \lambda_t + \zeta_{t+1}, \quad y_{t+1} = \sigma\left(mpk_{t+1}\right), r_{HML,t+1} ,$$

[22]Expected returns are computed using a standard two-stage approach – first, we estimate the betas from time-series regressions as described under prediction 1. We then measure expected returns as the predicted values from cross-sectional Fama-Macbeth regressions of returns on these betas. We provide further details in Appendix A.

[23]Our results are also robust to using a number of different asset pricing models to compute measures of beta and expected returns, such as the CAPM and Hou et al. (2015) investment-CAPM models. The relationship is robust to a variety of different controls and industry definitions as well. Finally, the results are qualitatively similar when we use the inter-quartile range instead of the standard deviation as our measure of within-industry dispersion.

Table 4: Industry-Level Dispersion in MPK, Expected Stock Returns and Beta

	(1)	(2)	(3)	(4)	(5)	(6)
$\sigma(\mathbb{E}[r])$	2.71***			1.20***		
	(30.11)			(9.82)		
$\sigma(\beta_{MKT})$		0.11***			0.08***	
		(6.48)			(3.31)	
$\sigma(\beta_{HML})$		0.14***			0.10***	
		(11.18)			(5.61)	
$\sigma(\beta_{SMB})$		0.14***			0.07***	
		(13.72)			(5.77)	
$\sigma(\beta_{CAPM,MPK})$			0.01***			0.09***
			(8.58)			(4.08)
$\sigma(\beta_{HML,MPK})$			0.06***			0.06***
			(7.96)			(4.80)
$\sigma(\beta_{SMB,MPK})$			0.06***			0.06***
			(10.38)			(5.70)
Observations	3203	3210	2398	3188	3194	2380
R^2	0.221	0.265	0.200	0.261	0.285	0.348
Industries	157	161	142	153	156	138
Year F.E.	No	No	No	Yes	Yes	Yes
Controls	No	No	No	Yes	Yes	Yes

Notes: This table reports a panel regression of the dispersion in *mpk* within industries on lagged measures of dispersion in risk exposure within those industries. An observation is an industry-year. $\mathbb{E}[r]$ is the expected return computed from the Fama-French model. β denotes the stock return beta on the Fama-French factors and β_{MPK} the *mpk* beta on the same factors. t-statistics are in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

where λ_t denotes the various proxies for the price of risk. Columns (1)-(3) of table 5 display regressions of the within-industry standard deviation of MPK, $\sigma(mpk_{t+1})$, on lagged measures of the PD ratio, GZ spread, and EB premium. All three measures significantly predict MPK dispersion, and in the direction the theory suggests: the GZ Spread and excess bond premium predict greater MPK dispersion, while a higher PD ratio predicts lower dispersion.[24] Because our measures of the price of risk are countercyclical, the results imply that variation in risk premia induce a countercyclical component in MPK dispersion, in line with (and potentially in part accounting for) the well known evidence of countercyclicality documented in Eisfeldt and Rampini (2006).[25]

Next, columns (4)-(6) of Table 5 report regressions of the cumulative twelve month return on the MPK-HML portfolio, $r_{HML,t+1}$ on the PD ratio, GZ spread, and EB premium. The GZ

[24]The results are qualitatively similar when we use a measure of unconditional (not industry-adjusted) MPK dispersion as the dependent variable.

[25]We report the correlations of these measures and de-trended GDP and TFP in Table 9 in Appendix A.2.

Table 5: Predictability of MPK Dispersion and MPK-HML Portfolio Return

	MPK Dispersion			MPK-HML Portfolio Returns		
	(1)	(2)	(3)	(4)	(5)	(6)
PD Ratio	-0.112***			-0.013*		
	(-3.52)			(-1.84)		
GZ Spread		1.226***			0.269**	
		(3.23)			(2.27)	
EB Premium			3.415***			0.426**
			(4.44)			(2.32)
Observations	166	166	166	166	166	166
R^2	0.103	0.072	0.185	0.031	0.082	0.068

Notes: This table reports time-series regressions of four-quarter ahead mpk dispersion and MPK-HML portfolio returns on measures of the price of risk. t-statistics are in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

spread and excess bond premium predict higher future returns on the MPK-HML portfolio, while the PD ratio predicts lower future returns, implying that a high price of risk predicts a greater excess return spread between high and low MPK firms. In sum, our findings confirm that indeed, investors require greater compensation (in the form of a higher rate of return) to invest in high MPK firms at times when risk premia are high, leading to a predictable and countercyclical increase in dispersion and widening of the spread between low and high MPK firms.[26]

4 Quantitative Model

In the next two sections, we use a more detailed version of the investment model laid out above to quantitatively investigate the contribution of heterogeneous risk premia to observed MPK dispersion. The model is kept deliberately simple in order to isolate the impact of our basic mechanism, namely dispersion in exposure to systematic risk. The theory consists of two main building blocks: (i) a stochastic discount factor, which we directly parameterize to be consistent with salient patterns in financial markets, i.e., high and countercyclical prices of risk, and (ii) a cross-section of heterogeneous firms, which make optimal investment decisions in the presence of firm-level and aggregate risk, given the stochastic discount factor. Specifying the stochastic discount factor exogenously allows us to sidetrack challenges with generating empirically relevant risk prices in general equilibrium, and focus on gauging the quantitative strength of our mechanism. To hone in on the effects of risk premia, we begin with a simplified

[26]De-trended GDP also predicts countercyclical MPK dispersion and return spreads between high and low MPK firms.

version in which we abstract from additional adjustment frictions. In this case, our framework yields exact closed form solutions for firm investment decisions. In Section 4.3, we extend the model to include capital adjustment costs. Our theoretical results there reveal an important amplification effect of these costs on the impact of risk premia.[27]

4.1 The Environment

Heterogeneity in risk exposures. The setup is a fleshed-out version of that in Section 2. We consider a discrete time, infinite-horizon economy. A continuum of firms of fixed measure one, indexed by i, produce a homogeneous good using capital and labor according to:

$$Y_{it} = X_t^{\hat{\beta}_i} \hat{Z}_{it} K_{it}^{\theta_1} N_{it}^{\theta_2}, \quad \theta_1 + \theta_2 < 1 \ .$$

Firm productivity (in logs) is equal to $\hat{\beta}_i x_t + \hat{z}_{it}$, where x_t denotes an aggregate component that is common across firms and $\hat{\beta}_i$ captures the exposure of the productivity of firm i to aggregate conditions.[28] We assume that $\hat{\beta}_i$ is distributed as $\hat{\beta}_i \sim \mathcal{N}\left(\bar{\hat{\beta}}, \sigma_{\hat{\beta}}^2\right)$ across firms. Heterogeneity in this exposure is a key ingredient of our framework – cross-sectional variation in $\hat{\beta}_i$ will lead directly to dispersion in expected MPK. The term $\hat{z}_{it}$ denotes a firm-specific, idiosyncratic component of productivity.

The two productivity components follow AR(1) processes (in logs):

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \\ \hat{z}_{it+1} &= \rho_z \hat{z}_{it} + \hat{\varepsilon}_{it+1}, \quad \hat{\varepsilon}_{it+1} \sim \mathcal{N}\left(0, \sigma_{\hat{\varepsilon}}^2\right) \ . \end{aligned} \tag{7}$$

Thus, there are two sources of uncertainty at the firm level – aggregate uncertainty, with conditional variance σ_ε^2, and idiosyncratic uncertainty, with variance $\hat{\sigma}_{\hat{\varepsilon}}^2$.

Stochastic discount factor. In line with the large literature on cross-sectional asset pricing in production economies, we parameterize directly the pricing kernel without explicitly modeling the consumer's problem. In particular, we specify the SDF as

$$\begin{aligned} \log M_{t+1} &\equiv m_{t+1} = \log \rho - \gamma_t \varepsilon_{t+1} - \frac{1}{2}\gamma_t^2 \sigma_\varepsilon^2 \\ \gamma_t &= \gamma_0 + \gamma_1 x_t \ , \end{aligned} \tag{8}$$

[27] We also consider the effects of other investment frictions, e.g., "wedges," or distortions, in Section 5.3.
[28] As above, we use lower-case to denote natural logs.

where ρ, $\gamma_0 > 0$ and $\gamma_1 \leq 0$ are constant parameters.[29] The SDF is determined by shocks to aggregate productivity. The conditional volatility of the SDF, given by $\sigma_m = \gamma_t \sigma_\varepsilon$, varies through time as determined by γ_t. This formulation allows us to capture in a simple manner a high, time-varying and countercyclical price of risk, as observed in the data (since $\gamma_1 < 0$, γ_t will be higher following economic contractions, i.e., when x_t is negative). Additionally, directly parameterizing γ_0 and γ_1 enables the model to be quantitatively consistent with key moments of asset returns, which are important for our analysis. The risk free rate is constant and equal to $-\log \rho$. Thus, γ_0 and γ_1 only affect the properties of equity returns., easing the interpretation of these parameters. The maximum attainable Sharpe ratio is equal to the conditional standard deviation of the SDF, i.e., $SR_t = \gamma_t \sigma_\varepsilon$ and the market price of risk is equal to the square of the Sharpe ratio, $\gamma_t^2 \sigma_\varepsilon^2$.

Input choices. Firms hire labor period-by-period after the realization of shocks at a competitive wage W_t. To keep the labor market simple, we assume that the equilibrium wage is given by

$$W_t = X_t^\omega \ ,$$

that is, the wage is a constant elasticity and increasing function of aggregate productivity, where $\omega \in [0, 1]$ determines the sensitivity of wages to aggregate conditions.[30] Maximizing over the static labor decision gives operating profits, i.e., revenues less labor costs, as

$$\Pi_{it} = G X_t^{\beta_i} Z_{it} K_{it}^\theta \ , \tag{9}$$

where $G \equiv (1 - \theta_2)\, \theta_2^{\frac{\theta_2}{1-\theta_2}}$, $\beta_i \equiv \frac{1}{1-\theta_2}\left(\hat{\beta}_i - \omega\theta_2\right)$, $Z_{it} \equiv \hat{Z}_{it}^{\frac{1}{1-\theta_2}}$ and $\theta \equiv \frac{\theta_1}{1-\theta_2}$. The exposure of firm profits to aggregate conditions is captured by β_i, which is a simple transformation of the underlying exposure of firm productivity to the aggregate component, $\hat{\beta}_i$, and the sensitivity of wages, ω.[31] The idiosyncratic component of productivity is similarly scaled, by $\frac{1}{1-\theta_2}$. The curvature of the profit function is equal to θ, which depends on the relative elasticities of capital and labor in production. These scalings reflect the leverage effects of labor liabilities on profits. From here on, we will primarily work with z_{it}, which has the same persistence as $\hat{z}_{it}$, i.e., ρ_z, and innovations $\varepsilon_{it+1} = \frac{1}{1-\theta_2}\hat{\varepsilon}_{t+1}$ with variance $\sigma_{\tilde{\varepsilon}}^2 = \left(\frac{1}{1-\theta_2}\right)^2 \sigma_{\hat{\varepsilon}}^2$. We will also use the fact that $\sigma_\beta^2 = \left(\frac{1}{1-\theta_2}\right)^2 \sigma_{\hat{\beta}}^2$. Notice that the profit function takes precisely the form assumed in Section 2. Thus, the firm's dynamic investment problem is given by expression (1).

[29] This specification builds closely on those in, for example, Zhang (2005) and Jones and Tuzel (2013).

[30] This setup follows, for example, Belo et al. (2014) and İmrohoroğlu and Tüzel (2014).

[31] The adjustment term for labor supply, $\omega\theta_2$, has a small effect on the mean of the β distribution, but otherwise does not affect our analysis.

Optimal investment. The simplicity of this setting leads to exact analytical expressions for the firm's investment decision. Specifically, we show in Appendix C.2.1 that the firm's optimal investment policy is given by:

$$k_{it+1} = \frac{1}{1-\theta}\left(\tilde{\alpha} + \beta_i \rho_x x_t + \rho_z z_{it} - \beta_i \gamma_t \sigma_\varepsilon^2\right) \,, \tag{10}$$

where $\tilde{\alpha} \equiv \log\theta + \log G - \alpha$, $\alpha \equiv r_f + \log\left(1 - (1-\delta)\,\rho\right)$ is a constant.[32] The firm's choice of capital is increasing in x_t and z_{it} due to their direct effect on expected future productivity (i.e., $\beta_i \rho_x x_t + \rho_z z_{it} = \mathbb{E}_t\left[\beta_i x_{t+1} + z_{it+1}\right]$), but, *ceteris paribus*, firms with higher betas choose a lower level of capital. The magnitude of this effect is larger when γ_t is large, i.e., in economic downturns. Clearly, with risk neutrality, i.e., $\gamma_0 = \gamma_1 = 0$, the last term is zero and investment is purely determined by expected productivity.

For a slightly different intuition, we substitute for γ_t and write the expression as

$$k_{it+1} = \frac{1}{1-\theta}\left(\tilde{\alpha} + \beta_i\left(\rho_x - \gamma_1\sigma_\varepsilon^2\right)x_t + \rho_z z_{it} - \beta_i \gamma_0 \sigma_\varepsilon^2\right) \,. \tag{11}$$

The risk premium affects the investment choice through both the time-varying and constant components of the price of risk: first, a more negative γ_1 increases the responsiveness of firms to aggregate conditions. Intuitively, a high (low) realization of x_t has two effects – first, since x_t is persistent, it signals that productivity is likely to be high (low) in the future, increasing (decreasing) investment (this force is captured by the ρ_x term). Moreover, a high (low) realization of x_t implies a low (high) price of risk, which further increases (decreases) investment. Second, the constant component of the risk premium, γ_0, adds a firm-specific constant – i.e., a firm fixed-effect – to the choice of capital, which leads to permanent dispersion in firm-level capital choices.

MPK dispersion. By definition, the realized *mpk* is given by $mpk_{it+1} = \log\theta + \pi_{it+1} - k_{it+1}$. Substituting for k_{it+1},

$$mpk_{it+1} = \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \gamma_t \sigma_\varepsilon^2 \,, \tag{12}$$

and taking conditional expectations,

$$Empk_{it+1} \equiv \mathbb{E}_t\left[mpk_{it+1}\right] = \alpha + \beta_i \gamma_t \sigma_\varepsilon^2 \,, \tag{13}$$

[32] More precisely, there are also terms that reflect the variance of shocks. Because these terms are negligible and play no role in our analysis (they are independent of the risk premium effects we measure), we suppress them here. The full expressions are given in Appendix C.2.1.

where α is as defined in equation (10) and reflects the risk-free user cost of capital. Expression (12) shows that dispersion in the realized *mpk* is due to both uncertainty over the realization of shocks, as well as the risk premium term, which is persistent at the firm level and depends on (i) the firm's exposure to the aggregate shock, β_i (and is increasing in β_i), and (ii) the time t price of risk, which is reflected in the term $\gamma_t \sigma_\varepsilon^2$. Intuitively, firm-level *mpk* deviations are composed of both a transitory component due to uncertainty and a persistent component due to the risk premium. The transitory components, however, are i.i.d. over time and thus lead to purely temporary deviations in *mpk* (this is true even though the underlying productivity processes are autocorrelated); the risk premium, on the other hand, leads to persistent deviations, in which firms that are more exposed to aggregate shocks, and so are riskier, will feature persistently high *mpk* deviations.

Expression (13) hones in on this second force and shows the persistent effects of risk premia on the conditional expectation of time $t+1$ *mpk*, denoted *Empk*. Indeed, in this simple case, the ranking of firms' *mpk* will be constant in expectation as determined by the risk premium – high beta firms will have permanently high *Empk* and low beta firms the opposite. Importantly, the *value* of *Empk* will fluctuate with γ_t, but the ordering across firms will be preserved. This is the sense that we call this component persistent/permanent. Expression (12) shows that this ordering will not be preserved period-by-period in terms of realized *mpk* – due to the realization of shocks, the ranking of firms' *mpk* deviations will fluctuate, but the firm-specific risk premium adds a persistent component.[33] Because the uncertainty portion of the realized *mpk* is always additively separable (in logs) and is independent of our mechanism, from here on we will primarily work with *Empk*.

Expression (14) presents the cross-sectional variance of *Empk*:

$$\sigma^2_{Empk_t} \equiv \sigma^2_{\mathbb{E}_t[mpk_{it+1}]} = \sigma^2_\beta \left(\gamma_t \sigma^2_\varepsilon\right)^2 \ . \tag{14}$$

Cross-sectional variation in *Empk* depends on the dispersion in beta and the price of risk. Dispersion will be greater when risk prices, reflected by $\gamma_t \sigma_\varepsilon^2$, are higher and so will be countercyclical. The average long-run level of *Empk* dispersion is given by

$$E\sigma^2_{Empk} \equiv \mathbb{E}\left[\sigma^2_{Empk_t}\right] = \sigma^2_\beta \left(\gamma_0^2 + \gamma_1^2 \sigma_x^2\right) \left(\sigma^2_\varepsilon\right)^2 \ , \tag{15}$$

where $\sigma_x^2 = \frac{\sigma_\varepsilon^2}{1-\rho_x^2}$.

An examination of expressions (13) and (14) confirms that the richer model here is consistent with the four key implications from Section 2, namely – (1) exposure to risk factors is a

[33]With additional adjustment frictions, there will be another factor confounding the relationship between beta and the realized and expected *mpk*.

determinant of *Empk*; (2) variation in the price of risk leads to predictable variation in mean *Empk*; (3) *mpk* dispersion is related to beta dispersion; and (4) *mpk* dispersion is increasing in the market price of risk, and so naturally contains a countercyclical element.

Aggregate outcomes. What are the implications of this dispersion in *Empk* for the aggregate economy? Our framework provides a natural way to quantify this by computing measures of aggregate productivity (TFP) and output. Appendix C.3 shows that aggregate output can be expressed as

$$\log Y_{t+1} \equiv y_{t+1} = a_{t+1} + \theta_1 k_{t+1} + \theta_2 n_{t+1} ,$$

where k_{t+1} denotes the aggregate capital stock, n_{t+1} aggregate labor and a_{t+1} the level of aggregate TFP, given by

$$a_{t+1} = a^*_{t+1} - \frac{1}{2}\frac{\theta_1 (1-\theta_2)}{1-\theta_1-\theta_2}\sigma^2_{mpk,t+1} , \tag{16}$$

where $\sigma^2_{mpk,t+1}$ is realized *mpk* dispersion in period $t+1$. The term a^*_{t+1} is the first-best level of TFP in the absence of any frictions (i.e., where marginal products are equalized). Thus, aggregate TFP monotonically decreases in the extent of capital "misallocation," captured by σ^2_{mpk}. The effect of misallocation on aggregate TFP depends on the overall curvature in the production function, $\theta_1+\theta_2$ and the relative shares of capital and labor. The higher is $\theta_1+\theta_2$, that is, the closer to constant returns to scale, the more severe the losses from mis-allocated resources. Similarly, fixing the degree of overall returns to scale, for a larger capital share, θ_1, a given degree of misallocation has larger effects on aggregate outcomes.

Using equation (14), the time t conditional expectation of one-period ahead TFP is given by

$$\mathbb{E}_t\left[a_{t+1}\right] = \mathbb{E}_t\left[a^*_{t+1}\right] - \frac{1}{2}\frac{\theta_1(1-\theta_2)}{1-\theta_1+\theta_2}\sigma^2_\beta\left(\gamma_t\sigma^2_\varepsilon\right)^2 . \tag{17}$$

Since γ_t is countercyclical, the expression shows that heterogeneity in risk premia lead to relatively lower (higher) levels of TFP during business cycle downturns (expansions). Taking expectations gives the effects on the average long-run level of productivity in the economy:

$$\bar{a} \equiv \mathbb{E}\left[\mathbb{E}_t\left[a_{t+1}\right]\right] = a^* - \frac{1}{2}\frac{\theta_1(1-\theta_2)}{1-\theta_1+\theta_2}\sigma^2_\beta\left(\gamma_0^2+\gamma_1^2\sigma_x^2\right)\left(\sigma^2_\varepsilon\right)^2 . \tag{18}$$

The expression directly links the extent of cross-sectional dispersion in required rates of return (which are in turn determined by the prices of risk and volatility of aggregate shocks) to the long-run level of aggregate productivity and gives a natural way to quantify the implications of these effects. In Sections 6.1 and 6.2, we show that our model can be extended to include multiple risk factors and to allow γ_t to depend on additional factors beyond the state of technology and so expressions (17) and (18) provide a more general connection between financial conditions (that may be less than perfectly correlated with the real economy) and aggregate productivity. Thus, these expressions provide one way to link the rich findings of the literature on cross-sectional asset pricing to real allocations and measures of aggregate performance. Further, they provide a new connection between aggregate volatility, e.g., the properties of the business cycle, and long-run economic outcomes.

4.2 The Cross-Section of Expected Stock Returns and MPK

In this section, we derive a sharp link between a firm's beta – and so its expected *mpk* – and its expected stock market return. This connection points to a novel empirical strategy to measure the dispersion in betas and so to quantify the *mpk* dispersion that arises from risk considerations using stock market data. Our key finding in this section is that, to a first-order approximation, the firm's expected stock return is a linear (and increasing) function of its beta.[34] Indeed, in the simple version of our model outlined thus far, the firm's expected *mpk* is proportional to its expected stock return. This link, first, justifies our use of data on expected stock returns and stock market betas as a proxy for expected *mpk* in Section 3 and second, shows that the dispersion in expected stock returns puts tight empirical discipline on the dispersion in betas and so expected *mpk* arising from risk channels – indeed, under some circumstances, they are proportional to one another. We use this connection to provide transparent intuition for our numerical approach in Section 5.

We obtain an analytic approximation for expected stock market returns by log-linearizing around the non-stochastic steady state where $X_t = Z_t = 1$. To a first-order, the (log of the) expected excess stock return of firm i (over the risk-free rate) is equal to (derivations in Appendix C.4)

$$Er^e_{it+1} \equiv \log \mathbb{E}_t \left[R^e_{it+1}\right] = \psi \beta_i \gamma_t \sigma^2_\varepsilon \ , \tag{19}$$

where

$$\psi = \frac{\frac{1}{\rho} + \delta - 1}{\frac{1}{\rho} + \delta \left(1 - \theta\right) - 1} \frac{1 - \rho}{1 - \rho\rho_x + \rho\gamma_1\sigma^2_\varepsilon} \ .$$

The expected excess return depends on the firm's beta (indeed, is linear and increasing in beta) and the price of risk. The risk premium is increasing in γ_0, and as γ_1 becomes more negative. Further, because the price of risk is countercyclical, risk premia increase during downturns for all firms and fall during expansions.[35] The time t cross-sectional dispersion in expected excess

[34] It is well known that a first-order approximation may not be sufficient to capture risk premia. In our quantitative work in Section 5, we work with numerical higher order approximations.

[35] Strictly speaking, these results hold in the approximation so long as $1 - \rho\rho_x + \rho\gamma_1\sigma^2_\varepsilon > 0$. This condition does not play a role in the numerical solution.

returns is given by

$$\sigma^2_{Er^e_t} \equiv \sigma^2_{\log \mathbb{E}_t\left[R^e_{it+1}\right]} = \psi^2 \sigma^2_\beta \left(\gamma_t \sigma^2_\varepsilon\right)^2 \ . \tag{20}$$

Similar to our findings for expected mpk, the expression reveals a tight link between beta dispersion and expected stock return dispersion. Indeed, were firms homogeneous with respect to their loadings on aggregate conditions, dispersion in expected returns would be zero. Moreover, as with expected mpk dispersion, expected stock return dispersion is increasing in the price of risk and so is countercyclical.

Comparing equations (13) and (19) shows that, in this setting, expected excess returns, Er^e_{it+1}, are proportional to expected mpk, $Empk_{it+1}$ and equations (14) and (20) show that $\sigma^2_{Er^e_t}$ is proportional to $\sigma^2_{Empk_t}$. Thus, the expressions reveal a tight connection between cross-sectional variation in expected stock returns and expected mpk – both are dependent on the variation in betas. Although the exact proportionality will not hold precisely in the full nonlinear model, we will use this intuition to quantify the role of risk considerations in generating dispersion in expected mpk.

Specifically, these results suggest an empirical strategy to estimate the three key structural parameters – γ_0, γ_1 and σ^2_β – using readily available stock market data. First, it is straightforward to verify that the market index – i.e., a perfectly diversified portfolio with no idiosyncratic risk – achieves the maximal Sharpe ratio:[36]

$$SR_{mt} = \gamma_t \sigma_\varepsilon, \qquad ESR_m \equiv \mathbb{E}\left[SR_{mt}\right] = \gamma_0 \sigma_\varepsilon \ . \tag{21}$$

The expression reveals a tight connection between the market Sharpe ratio and the parameter γ_0. Indeed, in this linearized environment, the mapping is one-to-one (given σ^2_ε). Next, deriving equation (19) for the market index gives

$$Er_{mt+1} = \psi \bar{\beta} \gamma_t \sigma^2_\varepsilon, \qquad Er_m \equiv \mathbb{E}\left[Er_{mt+1}\right] = \psi \bar{\beta} \gamma_0 \sigma^2_\varepsilon \ , \tag{22}$$

which shows that, given a value for γ_0, the market equity premium is increasing as γ_1 becomes more negative through its effects on ψ ($\bar{\beta}$ denotes the mean beta across firms). Lastly, equation (20) connects dispersion in beta, σ^2_β to dispersion in expected returns.

Together then, equations (20), (21) and (22) tightly link three observable moments of asset

[36]The Sharpe ratio for an individual firm is $SR_{it} = \frac{\beta_i \gamma_t \sigma^2_\varepsilon}{\sqrt{\left(\frac{1-\rho\rho_x+\rho\gamma_1\sigma^2_\varepsilon}{1-\rho\rho_z}\right)^2 \sigma^2_{\tilde{\varepsilon}} + \beta^2_i \sigma^2_\varepsilon}}$, which highlights that, due to the presence of idiosyncratic risk, individual firms do not attain the maximum Sharpe ratio. However, in this linear environment, the diversified index faces no risk from $\sigma^2_{\tilde{\varepsilon}}$, so that the expression collapses to (21). Although in the full numerical solution the market may not exactly attain this value due to the nonlinear effects of idiosyncratic shocks, the expression highlights that the market Sharpe ratio is informative about γ_0.

market data to the three parameters, γ_0, γ_1 and σ^2_β.

4.3 Adjustment Costs

In this section, we extend our framework to include capital adjustment costs. Although the main insights from the previous sections go through, we illustrate an important interaction between these costs and the effects of risk premia, namely, that adjustment frictions can amplify the impact of these systematic risk exposures on expected *mpk* dispersion.

We assume that capital investment is subject to quadratic adjustment costs, given by

$$\Phi\left(I_{it}, K_{it}\right) = \frac{\xi}{2}\left(\frac{I_{it}}{K_{it}} - \delta\right)^2 K_{it} \ .$$

In the presence of these costs, exact analytic solutions are no longer available. However, Appendix C.2.2 sets up the firm's problem and shows that to a first-order approximation, the firm's policy function is

$$k_{it+1} = \phi_{00} + \phi_1 \beta_i x_t + \phi_2 z_{it} + \phi_3 k_{it} - \phi_{01}\beta_i \ , \tag{23}$$

where

$$\begin{aligned} 0 &= \Big((\theta - 1) - \hat{\xi}\left(1+\rho\right)\Big)\phi_3 + \rho\hat{\xi}\phi_3^2 + \hat{\xi} \\ \phi_1 &= \frac{\left(\rho_x - \gamma_1\sigma^2_\varepsilon\right)\phi_3}{\hat{\xi}\left(1 - \rho\rho_x\phi_3 + \rho\gamma_1\sigma^2_\varepsilon\phi_3\right)}, \qquad \phi_2 = \frac{\rho_z\phi_3}{\hat{\xi}\left(1-\rho\rho_z\phi_3\right)} \\ \phi_{01} &= \frac{\phi_3}{\hat{\xi}\left(1-\rho\phi_3\right)}\frac{\gamma_0\sigma^2_\varepsilon}{1-\rho\rho_x\phi_3 + \rho\gamma_1\sigma^2_\varepsilon\phi_3} \ . \end{aligned}$$

We define the constant ϕ_{00} in the Appendix.[37] The term $\hat{\xi}$ is a composite parameter that captures the severity of adjustment costs, defined by $\hat{\xi} \equiv \frac{\xi}{1-\rho(1-\delta)}$.

Now, the past level of capital affects the new chosen level. The coefficient ϕ_3 captures the strength of this relationship. It lies between zero and one and is increasing in the adjustment cost, $\hat{\xi}$. It is independent of the risk premium. The other coefficients each have a counterpart in equation (11), but are modified to reflect the influence of adjustment costs. The coefficients ϕ_1 and ϕ_2 are both decreasing in the adjustment cost – intuitively, adjustment costs reduce the responsiveness to shocks. As adjustment costs tend to infinity, ϕ_3 approaches one and the latter two coefficients go to zero. As adjustment costs become large, the firm's choice of capital becomes more autocorrelated and eventually unresponsive to shocks. Importantly, ϕ_{01}

[37] As above, we ignore terms reflecting variance adjustments that are close to zero.

is increasing in these costs, showing that these additional adjustment frictions increase the importance of the firm's beta in determining its choice of capital.[38]

The expression for ϕ_{01} reveals an interaction between adjustment costs and time-varying risk – the denominator contains the product of ϕ_3 and γ_1, which implies that a more negative γ_1 leads to higher values of ϕ_{01} as long as adjustment costs are non-zero. Clearly this term disappears if adjustment costs are zero. In a moment, we will relate the value of ϕ_{01} to *Empk* dispersion. Thus, this interaction effect will increase the impact of risk premia on that dispersion.

In this setting, both risk premium effects and adjustment costs lead to *Empk* dispersion (realized *mpk* dispersion also depends on uncertainty, as above). Closed-form solutions are not available for period-by-period dispersion. However, to gain intuition, we are able to characterize the mean of firm-level expected *mpk* (which is also the mean of realized *mpk*, since the shocks are mean-zero) and thus the dispersion in this mean component:

$$\mathbb{E}\left[Empk_{it+1}\right] = -\frac{1-\theta}{1-\phi_3}\left(\phi_{00} - \phi_{01}\beta_i\right) \quad \Rightarrow \quad \sigma^2_{\mathbb{E}[Empk_{it+1}]} = \left(\frac{1-\theta}{1-\phi_3}\right)^2 \phi_{01}^2\sigma_\beta^2 \,. \tag{24}$$

Loosely speaking, the measure is the variance of the mean (i.e., permanent) component of firm-level *mpk* deviations. Recall that on their own, heterogeneous risk exposures only lead to persistent *mpk* deviations (in terms of the ordering across firms). These are exactly the effects we are picking up in (24).[39] Further, we are particularly interested in this component, since the data show an important role for a highly persistent (if not permanent) component in firm-level *mpk*. Notice also that ϕ_{01} is multiplicative in γ_0; in the absence of risk effects, there is no persistent *Empk* dispersion, even in the presence of adjustment costs.

Thus, expression (24) shows that the extended model continues to give rise to *mpk* deviations that are persistent at the firm-level. Moreover, the expression reveals a second amplification effect of adjustment costs through the $1-\phi_3$ term in the denominator. Recall that ϕ_3 is increasing in these costs, as is ϕ_{01}, so that holding fixed the other parameters, higher adjustment costs unambiguously increase risk effects on dispersion in *Empk*. An interesting implication of this result is that, perhaps surprisingly, adjustment frictions do not only affect transitory dispersion in the *mpk*. While this is true on their own, in conjunction with a fixed component in

[38]Strictly speaking, this is true so long as $1-\rho\rho_x\phi_3+\rho\gamma_1\sigma_\varepsilon^2\phi_3>0$. This condition holds for any reasonable level of adjustment costs, for example, given our estimates of the other parameters, ξ must be less than approximately 2180.

[39]Due to the interaction with adjustment costs, it is possible that these exposures can add to/subtract from the transitory dispersion created by those costs. We have derived an alternative measure of *Empk* dispersion, namely, the mean of the variance, defined as $\mathbb{E}\left[\sigma^2_{Empk}\right]$ (in contrast, equation (24) computes the variance of the mean). Quantitatively, we find that the contribution of risk exposures under this alternative is almost exactly that in expression (24). Because of nonlinearities, for that exercise, we calculate the portion due to risk premia effects as the total minus the implied dispersion when $\gamma_0=\gamma_1=0$.

the mpk, which we have here, these frictions can serve to amplify the effects of that component.

Finally, how do adjustment costs change the relationship between expected mpk, beta and expected stock returns? Appendix C.4 shows that to a first-order, expected returns are not affected by adjustment costs and so all the results from Section 4.2 continue to hold.[40] Thus, the arguments made in that section linking the key parameters of the model to moments of asset returns go through unchanged.

5 Quantitative Analysis

In this section, we use the analytical insights laid out above to numerically quantify the extent of mpk dispersion arising from risk premia effects.

5.1 Parameterization

We begin by assigning values to the more standard production parameters of our model. Following Atkeson and Kehoe (2005), we set the overall returns to scale in production $\theta_1 + \theta_2$ to 0.85. We assume standard shares for capital and labor of 0.33 and 0.67, respectively, which gives $\theta_1 = 0.28$ and $\theta_2 = 0.57$. These values imply $\theta = 0.65$.[41] We assume a period length of one year and accordingly set the rate of depreciation to $\delta = 0.08$. We estimate the adjustment cost parameter, ξ, in order to match the autocorrelation of investment, denoted $\text{corr}\left(\Delta k_t, \Delta k_{t-1}\right)$, which is 0.38 in our data. Equation (32) in Appendix C.5 provides a closed-form expression for this moment, which reveals a tight connection with the severity of adjustment frictions.[42]

To estimate the parameters governing the aggregate shock process, we build a long sample of Solow residuals for the US economy using data from the Bureau of Economic Analysis on real GDP and aggregate labor and capital. The construction of this series is standard (details in Appendix A.4). With these data, we use a standard autoregression to estimate the parameters ρ_x and σ_ε^2. This procedure gives values of 0.94 and 0.0247 for the two parameters, respectively.[43]

[40] Although this is only exactly true under our first-order approximation, Table 7 verifies numerically that adjustment costs have relatively modest effects on moments of returns.

[41] This is close to the values generally used in the literature. For example, Cooper and Haltiwanger (2006) estimate a value of 0.59 for US manufacturing firms. David and Venkateswaran (2017) use a value of 0.62.

[42] The expression also reveals that for ρ_x close to ρ_z, which we find in the data, described next, the autocorrelation of within-firm investment is almost invariant to the firm's beta (indeed, the invariance is exact if $\rho_x = \rho_z$). Thus, even with dispersion in betas, we may not see large variation in this moment across firms.

[43] The autoregression does not reject the presence of a unit root at standard confidence levels. We have also worked with the annual TFP series developed by John Fernald, available at `https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/`. These data are only available for the more recent post-war period, but also show that the series is close to a random walk (i.e., the autocorrelation of growth rates is essentially zero). To address possible concerns about aggregation affecting the stochastic properties of these series (i.e., persistence and volatility), we have also constructed an alternative series directly

Under our assumptions, firm-level productivity (including the aggregate component) can be measured directly (up to an additive constant) as $va_{it} - \theta k_{it}$ where va_{it} denotes the log of value added. After controlling for the level of aggregate productivity, a similar autoregression on the residual (firm-specific) component yields values for ρ_z and $\sigma_{\tilde{\varepsilon}}$ of 0.93 and 0.28, respectively.

Turning to the parameters of the SDF, we set $\rho = 0.988$ to match an average annual risk-free rate of 1.2%. Following the arguments in Section 4.2, we estimate the values of γ_0 and γ_1 to match the post-war (1947-2017) average annual excess return on the market index of 7.7% and Sharpe ratio of 0.53.[44] This strategy is equivalent to matching both the mean and volatility of market excess returns (the standard deviation is 14.6%). To be comparable to the data, stock returns in the model need to be adjusted for financial leverage. To do so, the mean and standard deviation of the model-implied returns need to be scaled by a factor of $1 + \frac{D}{E}$ where $\frac{D}{E}$ is the debt-to-equity ratio. We follow, e.g., Barro (2006) and assume an average debt-to-equity ratio of 0.5. Because both the numerator and denominator are scaled by the same constant, the Sharpe ratio is unaffected. For ease of interpretation, in what follows we report the properties of levered returns. To compute the return on the market, we must also take a stand on the mean beta across firms. Assuming that the mean of $\hat{\beta}_i$ (the underlying productivity beta) is one, and using the value of ω (the sensitivity of wages to aggregate shocks) suggested by İmrohoroğlu and Tüzel (2014) of 0.20, we can compute the mean beta to be 1.99.[45] This is simply the mean productivity beta adjusted for the leverage effects of labor liabilities. This procedure yields values of $\gamma_0 = 35$ and $\gamma_1 = -120$.

Finally, again following the insights in Section 4.2, we estimate the dispersion in betas to match the cross-sectional dispersion in expected stock returns. Because expected returns are not directly observable, we must choose an asset pricing model with which to estimate them. To be consistent with the broad literature, we use the expected returns predicted from the Fama and French (1992) 3-factor model as computed in Section 3.[46] The estimated average within-industry standard deviation of expected returns is 0.125 (we report more details and plot the full histogram of the expected return distribution in Appendix A.3: for example, the mean is about 12%, the 10th percentile about 0.3% and the 90th percentile about 24%. The interquartile range is just over 10%).[47] Feeding this value into our quantitative model (described

from the firm-level data by averaging across the firms in each year. This gives results quite similar to the baseline, $\rho_x = 0.92$ and $\sigma_\varepsilon = 0.0245$. Details are in Appendix A.4.

[44] We calculate these values using annualized monthly excess returns obtained from Kenneth French's website, `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

[45] İmrohoroğlu and Tüzel (2014) estimate this value to match the cyclicality of wages.

[46] See that section and Appendix A for details of this procedure.

[47] These values are quite close to the estimates in Lewellen (2015), who reports moments of the expected return distribution from a number of predictive models. For example, using annual data (although a slightly different time period and set of firms), he finds a standard deviation of up to 0.09, a mean of about 13% and 10th and 90th percentiles of 2% and 24%, respectively (e.g., Model 3, Panel A, Table 9b of that paper). Analyzing

next) yields an estimate for σ_β of 12, and adjusting for the scaling $1-\theta_2$ gives the dispersion in underlying productivity betas, $\sigma_{\hat{\beta}}$, equal to 5.00.

To accurately capture the properties of the time-varying risk premium, we solve for returns numerically using a fourth-order approximation in Dynare++. We describe the details of the numerical procedure in Appendix B. In brief, for a given set of parameters, we use the model solution to simulate time series of excess returns for a large panel of firms that differ in their betas. Averaging returns across these firms in each time period yields a series for the market return. We can then compute the mean and standard deviation (i.e., Sharpe ratio) of the market return. For each beta-type in each time period, we compute the expected excess return directly as the conditional expectation $\mathbb{E}_t\left[R^e_{it+1}\right]$ and then average over the time periods to obtain the average expected return for a firm of that beta-type. For this set of parameters, we also compute the autocorrelation of investment by applying equation (32). We then estimate the four parameters γ_0, γ_1, σ^2_β and ξ so that the results of this procedure leads to values of (i) market excess returns, (ii) market Sharpe ratio, (iii) cross-sectional dispersion in expected returns and (iv) the autocorrelation of investment that match the empirical values.

Table 6 summarizes our empirical approach/results.

Table 6: Parameterization - Summary

Parameter	Description	Value
Production		
θ_1	Capital share	0.28
θ_2	Labor share	0.57
δ	Depreciation rate	0.08
ξ	Adjustment cost	0.04
$\sigma_{\hat{\beta}}$	Std. dev. of risk exposures	5.00
Stochastic Processes		
ρ_x	Persistence of agg. shock	0.94
σ_ε	Std. dev. of agg. shock	0.0247
ρ_z	Persistence of idiosyncratic shock	0.93
$\sigma_{\tilde{\varepsilon}}$	Std. dev. of idiosyncratic shock	0.28
ω	Wage elasticity	0.20
Stochastic Discount Factor		
ρ	Time discount rate	0.988
γ_0	SDF – constant component	35
γ_1	SDF – time-varying component	-120

annualized monthly returns, he finds almost exactly the same value as do we for the standard deviation, namely, 0.125 (e.g., Model 3, Panel A, Table 3 of that paper). Although the values in Lewellen (2015) are not within-industry, this turns out to make only a small difference (we calculate the non-within industry standard deviation to be 0.132).

5.2 Risk-Based Dispersion in MPK

Table 7 presents our main quantitative results. We report four variants of our framework. The first column ("Baseline") corresponds to our full model with time-varying risk and adjustment costs. In the second column ("Only Risk"), we report the effects of risk premia without adjustment costs (i.e., ignoring the interaction effects demonstrated above). The third column ("Constant Risk") examines a version with adjustment costs but a constant price of risk (i.e., $\gamma_1 = 0$). The last column ("Only Constant Risk") has a constant price of risk and no adjustment costs. Our goal in showing these different permutations is to understand the role that each element of our model plays in leading to various patterns in *mpk* dispersion.

Table 7: Risk Premia and Misallocation

	Baseline (1)	Only Risk (2)	Constant Risk (3)	Only Constant Risk (4)
MPK Implications				
$E\sigma^2_{Empk}$	0.20	0.07	0.18	0.07
$\frac{E\sigma^2_{Empk}}{\sigma^2_{mpk}}$	44.4%	15.5%	39.8%	14.6%
$\frac{E\sigma^2_{Empk}}{\sigma^2_{\overline{mpk}}}$	66.7%	23.2%	59.6%	21.9%
$\Delta\bar{a}$	0.08	0.03	0.07	0.03
$\text{corr}\left(\sigma^2_{Empk_t}, x_t\right)$	−0.25	−0.98	0.48	0.08
Moments				
Er_m	0.08	0.10	0.05	0.06
ESR_m	0.53	0.67	0.55	0.72
$\text{corr}\left(\Delta k_t, \Delta k_{t-1}\right)$	0.38	−0.03	0.38	−0.03

Long-run effects. The first row of the table shows the average long-run level of *mpk* dispersion that stems from heterogeneous risk exposures. Once we have the estimated parameters, this is a simple application of expression (24), or without adjustment costs, the special case in (15). The second and third rows show the percentage of total observed misallocation that this value accounts for. In our sample, overall σ^2_{mpk} is 0.45. This is the denominator in the second row. Next, we compute the dispersion stemming from only the permanent component of observed misallocation. For each firm, we regress the time-series of its *mpk* deviations on a firm-level fixed effect. The fixed-effect is the permanent component of firm-level *mpk* and the residuals transitory components. We then compute the variance of the permanent component, which yields a value of $\sigma^2_{\overline{mpk}} = 0.30$, about two-thirds of the total. This is the denominator in the third row of the table. That row compares the *mpk* dispersion generated by risk effects, which is essentially all persistent in nature, to the permanent piece in the data. The next row quantifies the implications of the estimated dispersion for the long-run level of aggregate TFP. It reports the gains in TFP from eliminating this source of *mpk* dispersion, denoted $\Delta\bar{a}$.[48] This is just an application of expression (18).

Column (1) shows that in the full model, risk premium effects lead to *mpk* dispersion of 0.20. This accounts for about 44% of overall *mpk* dispersion in our data and about two-thirds of the permanent component. These values lead to a long-run TFP loss of about 8% (compared to an environment without risk premia, i.e., where $\gamma_0 = \gamma_1 = 0$). These results suggest that (i) variation in firm-level risk exposures can generate significant MPK dispersion, particularly when compared to the permanent component in the data, and (ii) the consequences for measures of aggregate performance such as TFP can be substantial. Column (2) shows that on their own, these exposures generate *mpk* dispersion of 0.07, which accounts for 15% of total σ^2_{mpk} in the data and for 23% of the permanent component. In other words, though the impacts of risk premia remain significant in isolation, they are less than half of those in column (1). These results highlight the important interaction with adjustment costs revealed in expression (24) – in the first column, these effects are taken into account; in the second column, they are not. The associated TFP losses are also smaller, but remain significant – the long-run level of TFP would be 3% higher without these effects.

Columns (3) and (4) show that the majority of these effects stems from the presence of a high persistent component in the price of risk, i.e., γ_0, rather than from the time-variation from γ_1. Setting $\gamma_1 = 0$ only modestly reduces the size of these effects in the presence of adjustment costs (compare columns (1) and (3)) and has a negligible effect on the results without them (columns (2) vs. (4)). The implication is that time-varying prices of risk do not add much to the average long-run level of *mpk* dispersion.

Time-variation. The last row in the top panel examines the second main implication of the theory, namely, the countercyclicality of *mpk* dispersion, which we measure as the correlation of $\sigma^2_{Empk_t}$ with the state of the business cycle, i.e., x_t.[49] Column (1) shows that the full model generates significantly countercyclical dispersion in *Empk* – the correlation of $\sigma^2_{Empk_t}$ with the state of the cycle is -0.25. To put this figure in context, Table 9 in Appendix A.2 shows that the correlation between σ^2_{mpk} and the cyclical component of aggregate productivity in the data is -0.27. Thus, our quantitative model predicts countercyclical dispersion on par with this value. Column (2) shows that as the only factor behind *Empk* dispersion, the time-varying

[48] Note that this calculation does not mean that policies eliminating this source of *mpk* dispersion here would necessarily be desirable. We merely see this as a useful way to quantify the implications of our findings.

[49] As noted above, with adjustment costs, we do not have analytic expressions for period-by-period *Empk* dispersion. We compute these values using simulation. Without adjustment costs, we can use expression (14) directly.

nature of risk premia would lead to an almost perfectly negative correlation with the business cycle. This is a clear implication of equation (14). The presence of adjustment costs in the first column confounds this relationship and leads to a smaller correlation (in absolute value) that is in line with the data. Finally, the last two columns illustrate that time-varying risk is key to generating countercyclical dispersion. Without this element, *Empk* dispersion is actually positive (significantly so with adjustment costs and mildly so without). Thus, our findings suggest that the interaction of a countercyclical price of risk with adjustment frictions is crucial in yielding a negative (though far from negative one) correlation between *Empk* dispersion and the state of the business cycle.

To highlight the potential implications of the countercyclical *Empk* dispersion produced by our model, consider the connection with the empirical results in Eisfeldt and Rampini (2006), who show that firm-level dispersion measures tend to be countercyclical, yet most capital reallocation is procyclical. Our theory can – at least in part – reconcile this observation due to the countercyclical nature of factor risk prices and the high beta of high MPK firms: countercyclical reallocation would entail moving capital to the riskiest of firms in the riskiest of times. Thus, in light of our results, it may not be as surprising that countercyclical dispersion obtains, even in a completely frictionless environment.

Moments. In the bottom panel of Table 7, we investigate the role that each element plays in matching the target moments. Our full model in column (1) is directly parameterized to match the three target moments, i.e., the equity premium, Sharpe ratio and autocorrelation of investment. In the second column, we show these moments from the version of our model without adjustment costs (i.e., setting $\xi = 0$ and the holding the other parameters at their estimated value). As implied by the approximation in Section 4.3, adjustment costs have a modest effect on the properties of returns (eliminating them raises the equity premium somewhat and the Sharpe ratio accordingly). However, the autocorrelation of investment falls dramatically without any adjustment friction, indeed, becoming slightly negative (due to the mean-reverting nature of shocks). Thus, some degree of adjustment costs is crucial for matching this latter moment. Comparing columns (1) and (3) shows that without time-varying risk, the model struggles to match the equity premium, which falls almost by half, from about 8% to 5%. As implied by expressions (21) and (32), time-varying risk has only modest effects on the average Sharpe ratio and the autocorrelation of investment. A similar pattern emerges from columns (2) and (4) – in the absence of adjustment costs, removing time-varying risk significantly reduces the equity premium but has little effect on the other two moments.

In sum, our results in Table 7 show first, ex-ante firm-level variation in risk exposures lead to quantitatively important dispersion in *mpk*; moreover, the dispersion from this source is persistent and can account for a significant portion of what seems to be a puzzling pattern in the data, namely, persistent mpk deviations at the firm-level. Second, these exposures add a notably countercyclical element to mpk dispersion, going some way towards reconciling the countercyclical nature of firm-level dispersion measures.

5.3 Other Distortions

Recent work has pointed to a number of additional factors (beyond fundamentals and technological adjustment costs) that may affect the firm's investment decisions and lead to mpk dispersion, including, for example, financial frictions, variable markups or policy-induced distortions. Moreover, it has been pointed out that attempts to identify one of these forces – while abstracting from others – may yield misleading conclusions. This section demonstrates that our strategy of using asset market data is robust to this critique. In other words, our approach yields accurate estimate of risk premia effects, even in the presence of other, un-modeled, distortions.

We first follow the broad literature, e.g., Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), and introduce these distortions as purely idiosyncratic "taxes" or "wedges" on firm revenues, $1 - e^{\tau_{it+1}}$ (so that the firm keeps a portion $e^{\tau_{it+1}}$). We work with the following specification for the wedge:

$$\tau_{it+1} = -\nu z_{it+1} - \eta_{it+1} \ . \tag{25}$$

The wedge is composed of two pieces. The first component is correlated with the firm's idiosyncratic productivity, where the strength of the relationship is captured by ν. If $\nu > 0$, the wedge discourages (encourages) investment by high (low) productivity firms. If $\nu < 0$, the opposite is true. The second component is uncorrelated with firm characteristics and can be either time-varying or fixed. Low (i.e., negative) values of η spur greater investment by firms irrespective of their underlying characteristics. We assume the firm knows the uncorrelated piece, η_{it+1}, when it chooses period t investment, i.e., k_{it+1}. Further, we assume that both components of the wedge are uncorrelated with the firm's beta. David and Venkateswaran (2017) show that this type of formulation can capture, for example, certain forms of financial frictions (due, e.g., to liquidity costs) and markups, in addition to policy-related distortions. We loosely refer to the wedge as a "distortion," although we do not take a stand on whether it stems from efficient factors or not, simply that there are other frictions in the allocation process. Appendix C.6 derives the following expression for the realized mpk:

$$mpk_{it+1} = \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \nu \rho_z z_{it} + \eta_{it+1} + \beta_i \gamma_t \sigma_\varepsilon^2 \ , \tag{26}$$

The distortion has several effects on the realized mpk. After the constant, the first two terms capture the effects of uncertainty over shocks and are identical to those in the baseline case. Next, the *mpk* includes a component that reflects the severity of the correlated distortion, ν, and depends on the firm's expectations of its idiosyncratic productivity ($\rho_z z_{it}$), leading to *mpk* deviations that are correlated with idiosyncratic productivity. Next, the *mpk* also depends on the uncorrelated component of distortions, η: firms with a high (positive) realization of η_{it+1} will invest less than their fundamentals would dictate, again leading to *mpk* deviations (that are uncorrelated with productivity). Finally, the last term reflects the risk premium, which, importantly, is independent of the distortions.

From expression (26), we can derive *Empk* and its cross-sectional variannce:

$$Empk_{it+1} = \alpha + \nu\rho_z z_{it} + \eta_{it+1} + \beta_i \gamma_t \sigma_\varepsilon^2, \quad \Rightarrow \quad \sigma^2_{Empk_t} = (\nu\rho_z)^2 \sigma_z^2 + \sigma_\eta^2 + \sigma_\beta^2 \left(\gamma_t \sigma_\varepsilon^2\right)^2 \ . \tag{27}$$

Dispersion in *Empk* is generated by three forces – first, the correlated component of the distortion, ν (its contribution to *mpk* dispersion also depends on the cross-sectional variance of expected idiosyncratic productivity, which is the term in parentheses); second, the variance of the uncorrelated component; and third, the variation in the risk premium.

Turning to stock market returns on the other hand, Appendix C.6 proves that equation (19) still holds. In other words, expected stock returns are independent of idiosyncratic distortions. These results imply that the mapping from expected returns to beta is, to a first-order, unaffected by these other distortions, as is the mapping from beta dispersion to its effects on *Empk*. Thus, even in the richer environment here, featuring the additional sources of misallocation revealed in expressions (26) and (27), using stock market data continues to yield accurate estimates of the effects of heterogeneous risk exposures alone.

Aggregate wedges. In principle, we can allow the wedge to also be correlated with aggregate productivity, x_t. Consider first the following formulation:

$$\tau_{it+1} = -\nu_z z_{it+1} - \nu_x x_{t+1} - \eta_{it+1} \ .$$

Here, the parameter ν_x captures the correlation of the distortion with the state of the business cycle. All firms are distorted by the aggregate component of the wedge, but all equally so. In this case, we can prove a similar result as with only idiosyncratic wedges – the distortion does not affect the cross-sectional dispersion in expected stock returns and so that moment still accurately pins down the relevant risk exposures (the wedge also does not affect the dispersion in *Empk* coming from risk premium effects).[50]

[50] The proofs for this section are in Appendix C.6.

As a second example, consider the following specification:

$$\tau_{it+1} = -\nu_z z_{it+1} - \nu_x \beta_i x_{t+1} - \eta_{it+1} \ .$$

Here, high beta firms are also disproportionately affected by the aggregate distortion. In this case, we can prove (Appendix C.6) that expected return dispersion gives a lower bound on risk premium effects if the wedge worsens in downturns, i.e., if $\gamma_x < 0$, which may be a plausible conjecture. On the other hand, we could be at risk of overstating these effects if the wedge worsens in expansions, i.e., $\gamma_x > 0$. However, it turns out that even in this case, our empirical results suggest a tight upper bound on the extent of the potential bias – specifically, the fact that *Empk* is countercyclical from prediction (2). To see this, Appendix C.6 derives the following expression for *Empk*:

$$\begin{aligned} \mathbb{E}_t \left[mpk_{it+1} \right] &= \alpha + \nu_z \rho_z z_{it} + \nu_x \beta_i \rho_x x_t + (1 - \nu_x) \beta_i \gamma_t \sigma_\varepsilon^2 + \eta_{it+1} \\ &= \alpha + \nu_z \rho_z z_{it} + \left(\nu_x \rho_x + (1 - \nu_x) \gamma_1 \sigma_\varepsilon^2 \right) \beta_i x_t + (1 - \nu_x) \beta_i \gamma_0 \sigma_\varepsilon^2 + \eta_{it+1} \ . \end{aligned}$$

The fact that *Empk* is countercyclical implies that the term in parentheses multiplying x_t should be negative, which puts the following bound on ν_x:

$$\frac{\nu_x}{1 - \nu_x} < - \frac{\gamma_1 \sigma_\varepsilon^2}{\rho_x} \ .$$

Intuitively, a positive value of ν_x adds a procyclical element to expected *mpk*. That *Empk* is actually countercyclical then puts a sharp bound on how large a positive value ν_x can take. Using the parameter estimates from Section 5, this calculation yields a maximum value of ν_x of about 0.065. In Appendix C.6 we derive an equation characterizing the potential bias in our estimate of *Empk* dispersion from a positive value of ν_x – even at this upper bound, the bias would be quantitatively negligible.

In sum, these results show that our empirical strategy yields accurate, and possibly conservative, estimates in the presence of other, possibly aggregate, factors distorting the firm's investment decisions. However, precisely pinning down the properties of such an aggregate wedge would likely involve taking a stand on its sources, e.g., time-varying financial frictions or markups, or policy-related distortions, and estimating its cyclical properties.

5.4 Alternative Sources of Heterogeneity

Variation in betas across firms is an essential ingredient in our theory. Our empirical approach measures these betas using dispersion in firm-level expected returns. Here, we explore whether other forms of firm-level heterogeneity can quantitatively generate the significant return dispersion observed in the data. In other words, we ask whether our estimates of beta dispersion are picking up meaningful dispersion from other potential sources.

First, we examine whether adjustments costs alone can generate substantial dispersion in conditional expected returns. To do so, we simulate a large panel of firms of a single beta-type (we set this to $\beta = 1.99$, which corresponds to the mean productivity beta of $\hat{\beta} = 1$). Although the firms are all of a single type, heterogeneity in conditional expected returns can arise from the presence of adjustment costs in combination with different histories of idiosyncratic shocks. The first column of Table 8 reports the results using the estimated value of ξ. The top row shows the minimum of the average of firm-level expected returns (i.e., we simulate a time series of expected returns for each firm, compute the long-run average for each firm and report the minimum), the second row the mean and the third row the maximum. The last row reports the spread between the minimum and the maximum. The estimated adjustment costs lead to very little dispersion in mean expected returns, e.g., the spread between the low and high firms is only about 0.3%.[51] To verify the robustness of this finding, column (2) repeats this analysis with a higher level of adjustment costs, namely, $\xi = 3$. The larger level of costs increases the level of expected returns slightly (recall that to a first-order, these costs should have no effects on the properties of expected returns) and has virtually no effect on the spread. Thus, it is unlikely that our estimates of beta are reflecting the effects of adjustment costs.

The remaining columns of Table 8 allow for variation in technological parameters across firms. Expression (19) provides some guidance as to the effects of some of these parameters on expected returns – taking derivatives, the expression implies that expected returns should be increasing in θ and δ. We also allow firms to differ in the properties of the stochastic process of idiosyncratic shocks, ρ_z and $\sigma^2_{\tilde{\varepsilon}}$. Although these do not influence expected returns under a first-order approximation, there may be effects due to the nonlinearities in the numerical model.

Column (3) examines heterogeneity in θ, the curvature of the profit function. Although there is little guidance on the extent of this heterogeneity (recall that all our estimations are within-industry), we compute expected returns for three values of θ, namely 0.85 (our baseline),

[51] This result should not be overly surprising – in the long run, the firms are identical, so mean expected returns should essentially be the same. We have also examined whether adjustment costs can lead to significant *transitory* dispersion in expected returns. To do so, we again simulate a large panel of firms with $\beta = 1.99$ and then compute period-by-period dispersion in expected returns. The mean of the cross-sectional standard deviation is 0.012 (and the maximum 0.057). This is relatively small compared to the observed standard deviation of expected returns of 0.125. Finally, we have also explored the effects of adjustment costs on the dispersion in *realized* returns. At the estimated level of adjustment costs, the mean standard deviation of realized stock returns for firms with the mean beta is 0.209. The corresponding value from the data (within-industry standard deviation of realized stock returns) is 0.789. The model-implied value increases only mildly with the higher adjustments costs, to 0.224, again suggesting that adjustment costs alone, while economically significant, do not generate substantial amounts of dispersion in these measures.

Table 8: Expected Return Dispersion – Other Forms of Heterogeneity

	Adjustment Costs					
	Estimated	Large	θ	δ	ρ_z	$\sigma^2_{\tilde{\varepsilon}}$
	(1)	(2)	(3)	(4)	(5)	(6)
Min. $\mathbb{E}[r_{it}]$	0.054	0.056	0.044	0.046	0.048	0.035
Mean $\mathbb{E}[r_{it}]$	0.055	0.058	0.055	0.055	0.055	0.055
Max. $\mathbb{E}[r_{it}]$	0.057	0.060	0.084	0.064	0.062	0.060
Spread	0.003	0.003	0.041	0.017	0.014	0.026

0.95 and 0.75.[52] In line with the predictions of expression (19), expected returns are increasing in θ. The first row reports the average expected return for a firm with low θ (0.75), the second row the baseline and the third row a high θ firm (0.95). The difference in mean expected returns between the highest and lowest θ firms is about 4%. This is an economically significant spread, suggesting that large differences in this parameter can result in meaningful differences in firm-level risk premia. However, even this substantial degree of heterogeneity cannot account for the even larger differences in expected returns observed in the data – for example, Table 10 in Appendix A.3 reports a spread between the 90th and 10th deciles of over 20%.

The last three columns show similar results for the remaining three parameters – the depreciation rate, δ, and persistence and volatility of idiosyncratic shocks, ρ_z and $\sigma^2_{\tilde{\varepsilon}}$. Expected returns are increasing in the first (as suggested by (19)) and decreasing in the other two. We examine values of ρ_z ranging from 0.50 to 0.95. For the other two parameters, we report the average expected return when doubling or halving their baseline values (the second row always reports the baseline).[53] Even for these large differences in parameter values, the predicted spread in expected returns only ranges from about 1.0% to 1.7%. Thus, a consistent message emerges across these experiments – unobserved variation in technological parameters seems unlikely to account for the large spreads in expected returns observed in the data.

5.5 Directly Measuring Productivity Betas

Our baseline approach to measuring firm-level risk exposures used the tight link between a firm's beta and its expected stock return laid out in Section 4.2. In this section, we use an alternative strategy to estimate the dispersion in these exposures using only production-side

[52]To put these values in context, they loosely correspond to the range of values found in the literature. For example, İmrohoroğlu and Tüzel (2014) use a value of 0.95. Clementi and Palazzo (2016) use 0.80.

[53]For $\sigma^2_{\tilde{\varepsilon}}$, we double or half the standard deviation, $\sigma_{\tilde{\varepsilon}}$, so the variance is scaled by a factor of four.

data. In one sense, this approach is more direct – there is no need to employ firm-level stock market data to measure risk exposures. On the other hand, computing betas directly from production-side data has its drawbacks – the data are of a lower frequency (quarterly at best) and the time dimension of the panel is shorter. Further, it may be difficult to apply this method to firms in developing countries (where measured misallocation tends to be larger), since most firm-level datasets there have relatively short panels and are at the annual frequency. For those reasons, we view our results here as an informative check on our baseline findings above.

The approach is as follows. For each firm, we regress measured productivity growth, i.e., $\Delta z_{it} + \beta_i \Delta x_t$ on aggregate productivity growth Δx_t. It is straightforward to verify that the coefficient from this regression is exactly equal to β_i. Using these estimates, we can compute the firm's underlying productivity $\hat{\beta}_i$ and simply calculate the cross-sectional dispersion in these estimates, $\sigma^2_{\hat{\beta}}$. We have applied this procedure using three different measures of the aggregate shock: (i) our long sample of Solow residuals, (ii) the series we construct from firm-level data (both of these are described in Section 5 and Appendix A.4) and (iii) the Fernald annual TFP series. The results yield values of $\sigma_{\hat{\beta}}$ of 6.4, 4.3 and 5.9, respectively. Recall that our estimate for this value using stock return data was 5.0, which is essentially the midpoint of the range found here.

6 Extensions

The framework we have outlined thus far features a tight connection between financial market conditions and the "real" side of the economy – indeed the state of technology, which is the single aggregate risk factor in the economy, determined both the common component of firm-level productivities and the price of risk simultaneously. In this section, we generalize that setup to allow for more flexible formulations of the determinants of financial conditions. Although empirically disciplining the additional factors added here may be challenging, we demonstrate that the same insights from our baseline setup go through.

6.1 A Multifactor Model

In principle, it is straightforward to include multiple aggregate risk factors in our setting. Here, we lay out a simple extension along these lines and show that analogous results hold (details in Appendix C.7). There are J factors. The profits of each firm has a vector of heterogeneous loadings on these factors, β_i, where the j-th element of β_i is the loading of firm i on factor j. The exposure of the SDF to the factors is captured by a vector of exposures γ, where element γ_j captures the exposure of the SDF to the j-th factor. For purposes of illustration, we assume

γ is constant through time and there are no adjustment costs (these are easily relaxed). The covariance matrix of factor innovations is given by Σ_f. The realized mpk is equal to

$$mpk_{it+1} = \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \Sigma_f \gamma' \ ,$$

where ε_{t+1} is the vector of shocks to these factors. Expected mpk and its cross-sectional dispersion are given by

$$\mathbb{E}_t \left[mpk_{it+1} \right] = \alpha + \beta_i \Sigma_f \gamma', \quad \sigma^2_{\mathbb{E}_t[mpk_{it+1}]} = \gamma \Sigma_f' \Sigma_\beta \Sigma_f \gamma' \ ,$$

where Σ_β is the covariance matrix of factor loadings across firms. This is the natural analog of expression (14): (i) expected mpk is determined by the firm's exposure to (all) the aggregate risk factors in the economy and the risk prices of those factors, and (ii) mpk dispersion is a function of the dispersion in those exposures across firms as captured by Σ_β.

Next, we can derive the following approximations for expected excess stock returns and the cross-sectional dispersion in expected returns:

$$\log \mathbb{E}_t \left[R^e_{it+1} \right] = \beta_i \psi \Sigma_f \gamma', \quad \sigma^2_{\log \mathbb{E}_t \left[R^e_{it+1} \right]} = \gamma \Sigma_f' \psi' \Sigma_\beta \psi \Sigma_f \gamma' \ ,$$

where ψ is a diagonal matrix with

$$\psi_{jj} = \frac{\frac{1}{\rho} + \delta - 1}{\frac{1}{\rho} + (1 - \theta)\,\delta - 1} \frac{1 - \rho}{1 - \rho \rho_j} \ ,$$

where ρ_j denotes the persistence of factor j. These are the analogs of expressions (19) and (20) – expected returns depend on factor exposures and the risk prices of those factors. Expected return dispersion depends on the dispersion in those exposures, here captured by Σ_β.

Thus, the same insights from our single factor model go through – dispersion in $Empk$ and expected returns are both determined by variation in exposures to the set of aggregate factors and hence, there is a tight relationship between the two. and To quantify the impact of these factors on mpk dispersion, however, we would need to know all the primitives governing the dynamics of the factors, e.g., the vector of persistences ρ and the covariance matrix Σ_f, and exposures, i.e., the exposures of the SDF, γ, and the vectors of firm loadings, Σ_β. This would likely entail taking a stand on the nature of each factor, computing their properties from the data and calibrating/estimating the γ vector and the covariance matrix of firm exposures, Σ_β.

6.2 Financial Shocks

Our baseline model tightly links financial conditions, for example, the price of risk, to macroeconomic conditions, i.e., the state of aggregate technology. However, financial conditions may not comove one-for-one with the "real" business cycle. It is straightforward to extend our setup to include pure financial shocks. Consider the following extension. The stochastic discount factor takes the form

$$\begin{aligned} m_{t+1} &= \log\rho - \gamma_t \varepsilon_{t+1} - \frac{1}{2}\gamma_t^2 \sigma_\varepsilon^2 \\ \gamma_t &= \gamma_0 + \gamma_f f_t , \end{aligned} \tag{28}$$

where

$$f_{t+1} = \rho_f f_t + \varepsilon_f, \quad \varepsilon_f \sim \mathcal{N}\left(0, \sigma_{\varepsilon_f}^2\right) .$$

In this formulation, f_t denotes the time-varying state of financial conditions, which is now disconnected from the state of aggregate technology. These financial factors may be correlated with real conditions, x_t, but need not be perfectly so. Thus, there is scope for changes in financial conditions, independent of those in real conditions, to affect the price of risk and through this channel, the allocation of capital.[54] Note also the difference between this setup and the one in Section 6.1 – here, the financial factor, f_t does not directly enter the profit function of the firm, it only affects the price of risk. Thus, it is a shock purely to financial market conditions. In contrast, the factors considered in Section 6.1 directly affected firm profitability.

Keeping the remainder of the environment the same as Section 4, we can derive exactly the same expressions for expected *mpk* and its cross-sectional variance, i.e.,

$$\mathbb{E}_t\left[mpk_{it+1}\right] = \alpha + \beta_i \gamma_t \sigma_\varepsilon^2, \quad \sigma^2_{\mathbb{E}_t[mpk_{it+1}]} = \sigma_\beta^2 \left(\gamma_t \sigma_\varepsilon^2\right)^2 ,$$

where now γ_t is a function of financial market conditions. When credit market conditions tighten (i.e., when f_t is small/negative since $\gamma_f < 0$), γ_t is high and *mpk* dispersion will rise. Finally, the average long-run level of *Empk* dispersion and aggregate productivity are given by

$$\mathbb{E}\left[\sigma^2_{Empk_t}\right] = \sigma_\beta^2 \left(\gamma_0^2 + \gamma_f^2 \sigma_f^2\right)\left(\sigma_{\varepsilon_f}^2\right)^2, \quad \bar{a} = a^* - \frac{1}{2}\frac{\theta_1\left(1-\theta_2\right)}{1-\theta_1-\theta_2}\sigma_\beta^2\left(\gamma_0^2 + \gamma_f^2 \sigma_f^2\right)\left(\sigma_{\varepsilon_f}^2\right)^2 ,$$

where $\sigma_f^2 = \frac{\sigma^2_{\varepsilon_f}}{1-\rho_f^2}$. The expressions reveal a tight connection between financial conditions and long-run performance of the economy – higher financial volatility ($\sigma^2_{\varepsilon_f}$), even independent of

[54]Our baseline model is the nested case where $\gamma_f = \gamma_1$ and f_t and x_t are perfectly correlated.

the state of the macroeconomy, induces greater persistent MPK dispersion and depresses the average level of achieved productivity.

7 Conclusion

In this paper, we have revisited the notion of "misallocation" from the perspective of a risk-sensitive, or risk-adjusted, version of the stochastic growth model with heterogeneous firms. The standard optimality condition for investment in this framework suggests that expected firm-level marginal products should reflect exposure to factor risks, and their pricing. To the extent that firms are differentially exposed to these risks, as the literature on cross-sectional asset pricing suggests, the implication is that cross-sectional dispersion in MPK may not only reflect true misallocation, but also risk-adjusted capital allocation. We provide empirical support for this proposition and demonstrate that a suitably calibrated model of firm-level investment behavior suggests that, indeed, risk-adjusted capital allocation accounts for a substantial fraction of observed MPK dispersion among US firms. Importantly, the majority of this dispersion is persistent in nature, which speaks to the large portion of observed MPK dispersion that arises from seemingly persistent/permanent factors at the firm-level. Further, our setup leads to a novel link between cross-sectional asset pricing, aggregate volatility and long-run productivity – our results suggest that there can be substantial "productivity costs" of business cycles.

There are several promising directions for future research. Our framework points to a new connection between business cycle dynamics and the cross-sectional allocation of inputs. Further investigation of this link, for example, a deeper exploration of the source of beta variation across firms, would lead to a better understanding of the underlying causes of observed marginal product dispersion across firms. The tractability of our setup allowed us to quantify the effects of financial market considerations, e.g., cross-sectional variation in required rates of return, on measures of economic performance, i.e., aggregate TFP. This link should be useful beyond the misallocation literature and provides a new way to evaluate the implications of the rich set of empirical findings in cross-sectional asset pricing. For example, pursuing multifactor extensions of our analysis (e.g., along the lines laid out in Sections 6.1 and 6.2) to incorporate the many risk factors pointed out in that literature would be fruitful to measure the implications of those factors for allocative efficiency and further assess the role of risk considerations in leading to misallocation. Of particular interest would be whether those factors are efficient or not, e.g., to what extent do capital allocations reflect the "mispricing" of assets.

References

Alvarez, F. and U. J. Jermann (2004): "Using asset prices to measure the cost of business cycles," *Journal of Political economy*, 112, 1223–1256.

Asker, J., A. Collard-Wexler, and J. De Loecker (2014): "Dynamic inputs and resource (mis) allocation," *Journal of Political Economy*, 122, 1013–1063.

Atkeson, A. and P. J. Kehoe (2005): "Modeling and measuring organization capital," *Journal of Political Economy*, 113, 1026–1053.

Balvers, R. J., L. Gu, D. Huang, M. Lee-Chin, et al. (2015): "Profitability, value and stock returns in production-based asset pricing without frictions," *Journal of Money, Credit, and Banking*.

Barro, R. J. (2006): "Rare disasters and asset markets in the twentieth century," *The Quarterly Journal of Economics*, 121, 823–866.

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013): "Cross Country Differences in Productivity: The Role of Allocative Efficiency," *American Economic Review*, 103, 305–334.

Belo, F., X. Lin, and S. Bazdresch (2014): "Labor hiring, investment, and stock return predictability in the cross section," *Journal of Political Economy*, 122, 129–177.

Binsbergen, J. V. and C. Opp (2017): "Real Anomalies," Wharton Working Paper.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101, 1964–2002.

Clementi, G. L. and B. Palazzo (2016): "Entry, exit, firm dynamics, and aggregate fluctuations," *American Economic Journal: Macroeconomics*, 8, 1–41.

Cochrane, J. (1991): "Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations," *Journal of Finance*, 46, 207–234.

Cooper, R. W. and J. C. Haltiwanger (2006): "On the nature of capital adjustment costs," *The Review of Economic Studies*, 73, 611–633.

David, J. M., E. Henriksen, and I. Simonovska (2014): "The risky capital of emerging markets," Tech. rep., National Bureau of Economic Research.

David, J. M., H. A. Hopenhayn, and V. Venkateswaran (2016): "Information, Misallocation and Aggregate Productivity," *The Quarterly Journal of Economics*, 131, 943–1005.

David, J. M. and V. Venkateswaran (2017): "The Sources of Capital Misallocation," Tech. rep., National Bureau of Economic Research.

Eisfeldt, A. and A. Rampini (2006): "Capital reallocation and liquidity," *Journal of Monetary Economics*, 53, 369–399.

Fama, E. F. and K. R. French (1992): "Cross-Section of Expected Stock Returns," *The Journal of Finance*, 47, 3247–3265.

Fama, E. F. and J. D. MacBeth (1973): "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy*, 81, 607–636.

Gilchrist, S. and E. Zakrajsek (2012): "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, 102, 1692–1720.

Gomes, J. and L. Schmid (2010): "Levered Returns," *Journal of Finance*, 65, 467–494.

Gomes, J., A. Yaron, and L. Zhang (2006): "Asset pricing implications of firms financing constraints," *Review of Financial Studies*, 19, 1321–1356.

Gopinath, G., Ş. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017): "Capital Allocation and Productivity in South Europe," *The Quarterly Journal of Economics*, 132, 1915–1967.

Hopenhayn, H. A. (2014): "Firms, misallocation, and aggregate productivity: A review," *Annu. Rev. Econ.*, 6, 735–770.

Hou, K., C. Xue, and L. Zhang (2015): "Digesting anomalies: An investment approach," *Review of Financial Studies*.

Hsieh, C. and P. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124, 1403–1448.

İmrohoroğlu, A. and Ş. Tüzel (2014): "Firm-level productivity, risk, and return," *Management Science*, 60, 2073–2090.

Jones, C. S. and S. Tuzel (2013): "Inventory Investment and The Cost of Capital," *Journal of Financial Economics*, 107, 557–579.

Kehrig, M. (2015): "The Cyclical Nature of the Productivity Distribution," *Working paper*.

Kehrig, M. and N. Vincent (2017): "Do Firms Mitigate or Magnify Capital Misallocation? Evidence from Planet-Level Data," *Working Paper*.

Lewellen, J. (2015): "The Cross-section of Expected Stock Returns," *Critical Finance Review*, 4, 1–44.

Lewellen, J. and S. Nagel (2006): "The conditional CAPM does not explain asset-pricing anomalies," *Journal of Financial Economics*, 82, 289–314.

Liu, L. X., T. M. Whited, and L. Zhang (2009): "Investment-based expected stock returns," *Journal of Political Economy*, 117, 1105–1139.

Lucas, R. E. (1987): *Models of Business Cycles*, vol. 26, Basil Blackwell Oxford.

Midrigan, V. and D. Y. Xu (2014): "Finance and misallocation: Evidence from plant-level data," *The American Economic Review*, 104, 422–458.

Moll, B. (2014): "Productivity losses from financial frictions: can self-financing undo capital misallocation?" *The American Economic Review*, 104, 3186–3221.

Peters, M. (2016): "Heterogeneous Mark-Ups, Growth and Endogenous Misallocation," *Working Paper*.

Restoy, F. and M. Rockinger (1994): "On Stock Market Returns and Returns on Investment," *Journal of Finance*, 49, 543–556.

Restuccia, D. and R. Rogerson (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments," *Review of Economic Dynamics*, 11, 707–720.

——— (2017): "The causes and costs of misallocation," *Journal of Economic Perspectives*, 31, 151–74.

Shiller, R. J. (2005): *Irrational Exuberance*, Princeton University Press, 2 ed.

Zhang, L. (2005): "The Value Premium," *Journal of Finance*, 60, 67–103.

——— (2017): "The Investment CAPM," *European Financial Management*.

Appendix

A Data

In this appendix, we describe the various data sources used throughout our analysis.

A.1 Compustat/CRSP

We obtain firm-level data from COMPUSTAT and CRSP. We include firms coded as industrial firms from 1965-2015. Our time-series regressions and portfolio sorts are run on a sample from 1973-2015, since data on the GZ spread and excess bond premium begins in 1973, and because there are relatively few industries with at least 10 firms in a given year pre-1973.[55] We further exclude financial firms by dropping those with COMPUSTAT SIC codes that correspond to finance, insurance, and real estate (FIRE, SIC codes 6000-6999). We also exclude firms with missing SIC codes or coded as non-classifiable, as much of our analysis examines within-industry variables. We measure firm revenue using sales from Compustat (series SALE), and capital using the depreciated value of plant, property, and equipment (series PPENT). We measure firm marginal product of capital in logs (up to an additive constant) as the difference between log revenue and capital, $mpk_{it} = y_{it} - k_{it}$. Market capitalization is measured as the price times shares outstanding from CRSP and profitability as the ratio of earnings before interest, taxes, depreciation, and amortization (EBITDA) divided by book assets (AT). We measure market leverage as the ratio of book debt to the sum of market capitalization plus book debt, where book debt is measured as current liabilities (LCT) + 1/2 long term debt (DLTT), following Gilchrist and Zakrajsek (2012). We measure book-to-market as the ratio of book equity to the market capitalization of the firm, where we measure book equity as the sum of shareholder's equity (SEQ), deferred taxes and investment credit (TXDITC) and the preferred stock liquidating value (PSTKL).

Computation of betas and expected returns. Here, we describe our procedure to compute stock market betas, MPK betas and expected returns.

We estimate stock market betas by performing time-series regressions of firm-level excess returns (realized returns from CRSP in excess of the risk-free rate), r^e_{it}, on aggregate factors, denoted by the $N \times 1$ vector $\mathbf{F_t}$. For each firm, the specification takes the form

$$r^e_{it} = \alpha_i + \beta_{it}\mathbf{F}_t + \epsilon_{it} \tag{29}$$

[55] Our portfolio sorts look qualitatively similar if we use data from the full 1965-2015 sample.

where β_{it} denotes the $1 \times N$ vector of period-t factor loadings. Under the CAPM, the single risk factor is the aggregate market return. Under the Fama-French 3 factor model, the risk factors are the market return (MKT), the return on a portfolio that is long in small firms and short in large ones (SMB) and the return on a portfolio that is long in high book-to-market firms and short on low ones (HML). We estimate these regressions (and the MPK betas described below) at the quarterly frequency using backwards-looking five-year rolling windows. We have also estimated the stock market betas using higher frequency monthly data (and two-year rolling windows) and obtained similar results.

To obtain a single measure of risk exposure from the multi-factor Fama-French model, we combine the resulting betas into a single value using estimated prices of risk from Fama and MacBeth (1973) regressions. Specifically, we estimate the following cross-sectional regression in each period:

$$r^e_{it} = \alpha_{it} + \lambda_t \beta_{it} + \epsilon_{it} \tag{30}$$

where λ_t denotes the $1 \times N$ vector of period t factor risk prices and β_{it} the $N \times 1$ vector of exposures, estimated as just described. We then calculate a single index of exposure to these factors as

$$\beta_{it,FF} = \lambda_t \beta_{it} = \sum_x \lambda_{t,x} \beta_{it,x}, \;\; x \in MKT, HML, SMB$$

We follow an analogous procedure to estimate MPK betas, simply replacing excess stock market returns on the left-hand side of (29) and (30) with mpk_{it}. The first regression yields measures of β_{MPK}, i.e., the exposure of each firm's MPK to the aggregate risk factors. The second regression combines these exposures into a single value in the multifactor model, using the coefficients from cross-sectional Fama and MacBeth (1973) regressions, which play the role of factor risk prices in determining the relationship between risk exposures and the cross-section of expected MPK.

Finally, we estimate expected stock returns as the predicted values from equation (30).

Composition-adjusted measures of mean and dispersion. For Predictions 2 and 4, we compute time-series of the mean and cross-sectional dispersion in MPK. Because Compustat is an unbalanced panel with significant changes in the composition of firms over time, it is important to ensure that we measure the variation in these objects due to changes in firm MPK, rather than additions or deletions from the dataset (especially since many additions and deletions to the Compustat data may not be true firm entry or exit). We therefore compute composition-adjusted measures of the mean and cross-sectional standard deviation in MPK that are only affected by firms who continue on in the dataset. We use the following procedure:

For each set of adjacent periods, e.g., t and $t+1$, we compute the statistic of interest in each time period (i.e., mean or cross-sectional standard deviation) only for those firms that are present in the data in both periods. Taking the difference yields the change in the statistic from time t to $t+1$ that is due only to changes in the common set of firms. Completing this procedure yields time-series of changes in the mean and cross-sectional standard deviation of MPK. We then combine these time-series of changes with the initial values of the statistics of interest (across all firms in the initial period) to construct a synthetic series for each statistic, which is not affected by the changing composition of firms in the data.

A.2 Time-Series Correlations

Table 9 reports contemporaneous correlations between (within-industry) MPK dispersion and indicators of the price of risk and the business cycle.

Table 9: Correlations of MPK Dispersion, the Price of Risk and the Business Cycle

	MPK Dispersion	PD Ratio	GZ Spread	EB Premium	GDP	TFP
MPK Dispersion	1.00					
PD Ratio	-0.42	1.00				
GZ Spread	0.39	-0.51	1.00			
EB Premium	0.51	-0.57	0.68	1.00		
GDP	-0.53	0.46	-0.59	-0.66	1.00	
TFP	-0.27	0.43	-0.32	-0.44	0.70	1.00

Notes: This table reports time-series correlations of MPK dispersion, measures of the price of risk and the business cycle. MPK dispersion is measured as the within-industry standard deviation in mpk. The PD ratio is the aggregate stock market price/dividend ratio. The GZ spread and EB (excess bond) premium are measures of credit spreads. GDP is log GDP and TPF is log TFP. We extract the cyclical components of GDP, TFP and the PD ratio using a one-sided Hodrick-Prescott filter. All series are described in more detail in the main text and Appendix A. All data are quarterly and are from 1973-2015.

A.3 Expected Return Distribution

Table 10 reports statistics from the cross-sectional distribution of expected returns. As discussed in Section 5, the (average within-industry) standard deviation is 0.125. The mean is about 12%, the 10th and 90th percentiles about 0.3% and 24%, respectively, and the interquartile range (75th less 25th percentiles) just over 10%.

Table 10: The Distribution of Expected Returns

Percentile	10th	25th	Mean	75th	90th	Std. Dev.
Value	0.3%	7.0%	12.0%	18.0%	24.0%	12.5%

Figure 1 plots the full cross-sectional distribution of expected returns:

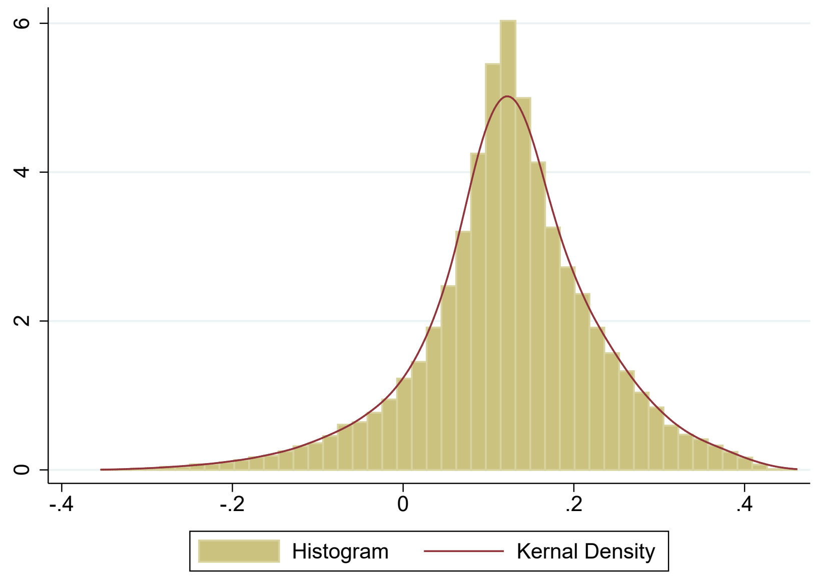

Figure 1: Cross-Sectional Distribution of Expected Returns

A.4 Aggregate Productivity Series

Solow residuals. To build a series of Solow residuals, we obtain data on real GDP and aggregate labor and capital from the Bureau of Economic Analysis. Data on real GDP are from BEA Table 1.1.3 ("Real Gross Domestic Product"), data on labor are from BEA Table 6.4 ("Full-Time and Part-Time Employees") and data on the capital stock are from BEA Table 1.2 ("Net Stock of Fixed Assets"). The data are available annually from 1929-2016. With these data we compute $x_t = y_t - \theta_1 k_t - \theta_2 n_t$. We extract a linear time-trend and then estimate the autoregression in equation (7).

Firm-level series. To construct the alternative series for aggregate productivity from the firm-level data, we use the following procedure. First, we compute firm-level productivity as $z_{it} + \beta_i x_t = y_{it} - \theta k_{it}$. We then average these values across all firms in each year. Because z_{it} is mean-zero and independent across firms, this yields a scaled measure of aggregate productivity, $\bar{\beta} x_t$, where $\bar{\beta}$ is the mean beta across firms, which under our assumptions, is approximately two. We extract a linear time-trend from this series and then estimate the autoregression. The coefficient from this regression gives ρ_x. The standard deviations of the residuals gives $\bar{\beta}\sigma_\varepsilon$ and after dividing by $\bar{\beta}$ gives the true volatility of shocks. Applying this procedure to the set of Compustat firms over the period 1962-2016 yields values of $\rho_x = 0.92$ and $\sigma_\varepsilon = .0245$.

B Numerical Procedure

Our numerical approach to parameterize the model is as follows. For a given set of the parameters γ_0, γ_1 and σ_β^2, we compute the autocorrelation using equation (32). We solve the model for a wide grid of beta-types centered around the mean beta. We use an 11 point grid ranging from -3 to 7 (the results are not overly sensitive to the width of the grid). We simulate a time series of excess returns for a large number of firms of each type. We then average the returns across firms in each time period, which yields a series for the market excess return, and compute the mean and standard deviation of this series.

Next we compute the expected return for each beta-type as the mean of the conditional expectation of returns, i.e., $\mathbb{E}_t\left[R_{it+1}\right] = \mathbb{E}_t\left[\frac{D_{it+1}+P_{it+1}}{P_{it}}\right]$. We then use these values to calculate the dispersion in expected returns, σ_{Er}^2, interpolating for values of β that are not on the grid. Finally, we find the set of the four parameters, γ_0, γ_1, σ_β^2 and ξ that make the simulated moments consistent with the empirical ones, i.e., expected excess market returns, market Sharpe ratio, dispersion in expected returns and autocorrelation of investment. As noted in the text, we implement this procedure for returns using a fourth-order approximation in Dynare++.

C Derivations and Proofs

This appendix provides detailed derivations for the expressions in the text.

C.1 Motivation

Derivation of equation (3).

$$\begin{aligned} 1 &= \mathbb{E}_t\left[M_{t+1}\left(MPK_{it+1}+1-\delta\right)\right] \\ &= \mathbb{E}_t\left[M_{t+1}\right]\mathbb{E}_t\left[MPK_{it+1}+1-\delta\right] + \operatorname{cov}\left(M_{t+1}, MPK_{it+1}\right) \end{aligned}$$

Consider the MPK of a 'risk-free' firm defined by $\operatorname{cov}\left(M_{t+1}, MPK_{ft+1}\right) = 0$. We have

$$1 = \mathbb{E}_t\left[M_{t+1}\right]\left(MPK_{ft+1}+1-\delta\right)$$

and combining,

$$\begin{aligned} \mathbb{E}_t\left[MPK_{it+1}\right] &= MPK_{ft+1} - \frac{\operatorname{cov}\left(M_{t+1}, MPK_{it+1}\right)}{\mathbb{E}_t\left[M_{t+1}\right]} \\ &= \alpha_t + \beta_{it}\lambda_t \end{aligned}$$

where α_t, β_{it} and λ_t are as defined in the text.. By a no-arbitrage condition, it must be the case that $\frac{1}{\mathbb{E}_t[M_{t+1}]} = MPK_{ft+1} + 1 - \delta = R_{ft}$ where R_{ft} is the gross risk-free interest rate.

No aggregate risk. With no aggregate risk, $M_{t+1} = \rho \, \forall \, t$ where ρ is the rate of time discount. The Euler equation gives

$$1 = \rho \left(\mathbb{E}_t \left[MPK_{it+1}\right] + 1 - \delta\right) \; \forall \; i, t \quad \Rightarrow \quad \mathbb{E}_t \left[MPK_{it+1}\right] = \frac{1}{\rho} - (1-\delta) = r_f + \delta$$

CAPM. Clearly, $-\text{cov}\left(M_{t+1}, MPK_{it+1}\right) = b\text{cov}\left(R_{mt+1}, MPK_{it+1}\right)$ and $\text{var}\left(M_{t+1}\right) = b^2\text{var}\left(R_{mt+1}\right)$. Since the market return is an asset, it must satisfy $\mathbb{E}_t\left[R_{mt+1}\right] = R_{ft} + \frac{\lambda_t}{b}$ so that $\lambda_t = b\left(\mathbb{E}_t\left[R_{mt+1}\right] - R_{ft}\right)$. Substituting into expression (3) gives the CAPM expression in the text.

CCAPM. A log-linear approximation to the SDF around its unconditional mean gives $M_{t+1} \approx \mathbb{E}\left[M_{t+1}\right]\left(1 + m_{t+1} - \mathbb{E}\left[m_{t+1}\right]\right)$ and in the case of $CRRA$ utility, $m_{t+1} = -\gamma \Delta c_{t+1}$ where Δc_{t+1} is log consumption growth. Substituting for M_{t+1} into expression (3) gives the CCAPM expression in the text.

C.2 Model Solution

C.2.1 Baseline Environment

The static labor choice solves

$$\max e^{\hat{z}_{it} + \hat{\beta}_i x_t} K_{it}^{\theta_1} N_{it}^{\theta_2} - W_t N_{it}$$

with the associated first order condition

$$N_{it} = \left(\frac{\theta_2 e^{\hat{z}_{it} + \hat{\beta}_i x_t} K_{it}^{\theta_1}}{W_t}\right)^{\frac{1}{1-\theta_2}}$$

Substituting for the wage with $W_t = X_t^{\omega}$ and rearranging gives operating profits

$$\Pi_{it} = G e^{\beta_i x_t + z_{it}} K_{it}^{\theta}$$

where $G \equiv (1-\theta_2)\, \theta_2^{\frac{\theta_2}{1-\theta_2}}$, $\beta_i = \frac{1}{1-\theta_2}\left(\hat{\beta}_i - \omega\theta_2\right)$, $z_{it} = \frac{1}{1-\theta_2}\hat{z}_{it}$ and $\theta = \frac{\theta_1}{1-\theta_2}$, which is equation (9) in the text.

The first order and envelope conditions associated with (1) give the Euler equation:

$$\begin{aligned} 1 &= \mathbb{E}_t\left[M_{t+1}\left(\theta e^{z_{it+1}+\beta_i x_{t+1}} G K_{it+1}^{\theta-1}+1-\delta\right)\right] \\ &= (1-\delta)\,\mathbb{E}_t\left[M_{t+1}\right]+\theta G K_{it+1}^{\theta-1}\mathbb{E}_t\left[e^{m_{t+1}+z_{it+1}+\beta_i x_{t+1}}\right] \end{aligned}$$

Substituting for m_{t+1} and rearranging,

$$\begin{aligned} \mathbb{E}_t\left[e^{m_{t+1}+z_{it+1}+\beta_i x_{t+1}}\right] &= \mathbb{E}_t\left[e^{\log\rho-\gamma_t\varepsilon_{t+1}-\frac{1}{2}\gamma_t^2\sigma_\varepsilon^2+z_{it+1}+\beta_i x_{t+1}}\right] \\ &= \mathbb{E}_t\left[e^{\log\rho+\rho_z z_{it}+\varepsilon_{it+1}+\beta_i\rho_x x_t+(\beta_i-\gamma_t)\varepsilon_{t+1}-\frac{1}{2}\gamma_t^2\sigma_\varepsilon^2}\right] \\ &= e^{\log\rho+\rho_z z_{it}+\beta_i\rho_x x_t+\frac{1}{2}\sigma_{\tilde\varepsilon}^2+\frac{1}{2}\beta_i^2\sigma_\varepsilon^2-\beta_i\gamma_t\sigma_\varepsilon^2} \end{aligned}$$

and

$$\mathbb{E}_t\left[M_{t+1}\right]=\mathbb{E}_t\left[e^{\log\rho-\gamma_t\varepsilon_{t+1}-\frac{1}{2}\gamma_t^2\sigma_\varepsilon^2}\right]=e^{\log\rho+\frac{1}{2}\gamma_t^2\sigma_\varepsilon^2-\frac{1}{2}\gamma_t^2\sigma_\varepsilon^2}=\rho$$

so that

$$\theta G K_{it+1}^{\theta-1}=\frac{1-(1-\delta)\,\rho}{e^{\log\rho+\rho_z z_{it}+\beta_i\rho_x x_t+\frac{1}{2}\sigma_{\tilde\varepsilon}^2+\frac{1}{2}\beta_i^2\sigma_\varepsilon^2-\beta_i\gamma_t\sigma_\varepsilon^2}}$$

and rearranging and taking logs,

$$k_{it+1}=\frac{1}{1-\theta}\left(\tilde\alpha+\frac{1}{2}\sigma_{\tilde\varepsilon}^2+\frac{1}{2}\beta_i^2\sigma_\varepsilon^2+\rho_z z_{it}+\beta_i\rho_x x_t-\beta_i\gamma_t\sigma_\varepsilon^2\right)$$

where

$$\begin{aligned} \tilde\alpha &= \log\theta+\log G-\alpha \\ \alpha &= -\log\rho+\log\left(1-(1-\delta)\,\rho\right)=r_f+\log\left(1-(1-\delta)\,\rho\right) \end{aligned}$$

Ignoring the variance terms gives equation (10).

The realized mpk is given by

$$\begin{aligned} mpk_{it+1} &= \log\theta+\pi_{it+1}-k_{it+1} \\ &= \log\theta+\log G+z_{it+1}+\beta_i x_{t+1}-(1-\theta)\,k_{it+1} \\ &= \log\theta+\log G+z_{it+1}+\beta_i x_{t+1}-\tilde\alpha-\rho_z z_{it}-\beta_i\rho_x x_t+\beta_i\gamma_t\sigma_\varepsilon^2 \\ &= \alpha+\varepsilon_{it+1}+\beta_i\varepsilon_{t+1}+\beta_i\gamma_t\sigma_\varepsilon^2 \end{aligned}$$

The time t conditional expected mpk is

$$\mathbb{E}_t\left[mpk_{it+1}\right]=\alpha+\beta_i\gamma_t\sigma_\varepsilon^2$$

and the time t and mean cross-sectional variances are, respectively,

$$\begin{aligned}
\sigma^2_{\mathbb{E}_t[mpk_{it+1}]} &= \sigma^2_\beta \left(\gamma_t \sigma^2_\varepsilon\right)^2 \\
\mathbb{E}\left[\sigma^2_{\mathbb{E}_t[mpk_{it+1}]}\right] &= \mathbb{E}\left[\sigma^2_\beta \left(\gamma_0 + \gamma_1 x_t\right)^2 \left(\sigma^2_\varepsilon\right)^2\right] = \sigma^2_\beta \left(\gamma_0^2 + \gamma_1^2 \sigma_x^2\right)\left(\sigma^2_\varepsilon\right)^2
\end{aligned}$$

C.2.2 Adjustment Costs

With capital adjustment costs, the firm's investment problem takes the form

$$\begin{aligned}
V\left(X_t, Z_{it}, K_{it}\right) = \max_{K_{it+1}} \quad & G X_t^{\beta_i} Z_t K_{it}^\theta - K_{it+1} + (1-\delta) K_{it} - \Phi\left(I_{it}, K_{it}\right) \\
+ \quad & \mathbb{E}_t\left[M_{t+1} V\left(X_{t+1}, Z_{it+1}, K_{it+1}\right)\right]
\end{aligned} \tag{31}$$

Policy function. The first order and envelope conditions associated with (31) give the Euler equation:

$$\begin{aligned}
1 + \xi\left(\frac{K_{it+1}}{K_{it}} - 1\right) &= \mathbb{E}_t\left[M_{t+1}\left(G\theta e^{z_{it+1} + \beta_i x_{t+1}} K_{it+1}^{\theta-1} + 1 - \delta - \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}} - 1\right)^2 + \xi\left(\frac{K_{it+2}}{K_{it+1}} - 1\right)\frac{K_{it+2}}{K_{it+1}}\right)\right] \\
&= \mathbb{E}_t\left[M_{t+1}\left(G\theta e^{z_{it+1} + \beta_i x_{t+1}} K_{it+1}^{\theta-1} + 1 - \delta + \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}}\right)^2 - \frac{\xi}{2}\right)\right]
\end{aligned}$$

In the non-stochastic steady state,

$$\begin{aligned}
MPK &= G\theta K^{\theta-1} = \frac{1}{\rho} + \delta - 1 \quad \Rightarrow \quad K = \left[\frac{1}{G\theta}\left(\frac{1}{\rho} + \delta - 1\right)\right]^{\frac{1}{\theta-1}} \\
\Pi &= G K^\theta \quad \Rightarrow D = G K^\theta - \delta K \\
P &= \frac{\rho}{1-\rho} D \\
R &= 1 + \frac{D}{P} = \frac{1}{\rho} \quad \Rightarrow r_f = -\log \rho
\end{aligned}$$

Define the investment return:

$$R^I_{it+1} = \frac{G\theta e^{z_{it+1} + \beta_i x_{t+1}} K_{it+1}^{\theta-1} + 1 - \delta + \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}}\right)^2 - \frac{\xi}{2}}{1 + \xi\left(\frac{K_{it+1}}{K_{it}} - 1\right)}$$

and log-linearizing,

$$\begin{aligned}
r^I_{it+1} &= \rho G\theta K^{\theta-1}\left(z_{it+1} + \beta_i x_{t+1}\right) + \left(\rho G\theta(\theta-1) K^{\theta-1} - \xi(1+\rho)\right) k_{it+1} + \rho\xi k_{it+2} + \xi k_{it} \\
&- \log\rho - \rho G\theta(\theta-1) K^{\theta-1} k
\end{aligned}$$

where $k = \log K$.

To derive the investment policy function, conjecture it takes the form

$$k_{it+1} = \phi_{0i} + \phi_1 \beta_i x_t + \phi_2 z_{it} + \phi_3 k_{it}$$

Then,

$$k_{it+2} = \phi_{0i}\left(1+\phi_3\right) + \phi_1\beta_i\left(\rho_x+\phi_3\right)x_t + \phi_2\left(\rho_z+\phi_3\right)z_{it} + \phi_3^2 k_{it} + \phi_1\beta_i\varepsilon_{t+1} + \phi_2\varepsilon_{it+1}$$

Substituting into the investment return,

$$\begin{aligned}
r^I_{it+1} &= \left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1-\rho\phi_3\right)\right)\phi_{0i} - \log\rho - \rho G\theta\left(\theta-1\right)K^{\theta-1}k \\
&+ \left(\rho G\theta K^{\theta-1}\rho_z + \left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_2 + \rho\xi\left(\rho_z+\phi_3\right)\phi_2\right)z_{it} \\
&+ \left(\rho G\theta K^{\theta-1}\rho_x + \left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_1 + \rho\xi\left(\rho_x+\phi_3\right)\phi_1\right)\beta_i x_t \\
&+ \left(\left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_3 + \rho\xi\phi_3^2 + \xi\right)k_{it} \\
&+ \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_2\right)\varepsilon_{it+1} + \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\beta_i\varepsilon_{t+1}
\end{aligned}$$

and

$$\begin{aligned}
r^I_{it+1} + m_{it+1} &= \left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1-\rho\phi_3\right)\right)\phi_{0i} - \rho G\theta\left(\theta-1\right)K^{\theta-1}k - \frac{1}{2}\gamma_0^2\sigma_\varepsilon^2 - \frac{1}{2}\gamma_1^2\sigma_\varepsilon^2 x_t^2 \\
&+ \left(\rho G\theta K^{\theta-1}\rho_z + \left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_2 + \rho\xi\left(\rho_z+\phi_3\right)\phi_2\right)z_{it} \\
&+ \left(\left(\rho G\theta K^{\theta-1}\rho_x + \left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_1 + \rho\xi\left(\rho_x+\phi_3\right)\phi_1\right)\beta_i - \gamma_0\gamma_1\sigma_\varepsilon^2\right)x_t \\
&+ \left(\left(\rho G\theta\left(\theta-1\right)K^{\theta-1} - \xi\left(1+\rho\right)\right)\phi_3 + \rho\xi\phi_3^2 + \xi\right)k_{it} \\
&+ \left(\rho G\theta K^{\theta-1} + \rho\xi\phi_2\right)\varepsilon_{it+1} + \left(\left(\rho G\theta K^{\theta-1} + \rho\xi\phi_1\right)\beta_i - \gamma_0 - \gamma_1 x_t\right)\varepsilon_{t+1}
\end{aligned}$$

The Euler equation governing the investment return implies

$$
\begin{aligned}
0 &= \mathbb{E}_t\left[r^I_{it+1}+m_{t+1}\right]+\frac{1}{2}\mathrm{var}\left(r^I_{it+1}+m_{it+1}\right)\\
&= \left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1-\rho\phi_3\right)\right)\phi_{0i}-\rho G\theta\left(\theta-1\right)K^{\theta-1}k\\
&+ \left(\rho G\theta K^{\theta-1}\rho_z+\left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1+\rho\right)\right)\phi_2+\rho\xi\left(\rho_z+\phi_3\right)\phi_2\right)z_{it}\\
&+ \left(\rho G\theta K^{\theta-1}\rho_x+\left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1+\rho\right)\right)\phi_1+\rho\xi\left(\rho_x+\phi_3\right)\phi_1-\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_1\right)\gamma_1\sigma^2_\varepsilon\right)\beta_i x_t\\
&+ \left(\left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1+\rho\right)\right)\phi_3+\rho\xi\phi_3^2+\xi\right)k_{it}\\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_2\right)^2\sigma^2_{\tilde{\varepsilon}}\\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_1\right)^2\beta_i^2\sigma^2_\varepsilon-\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_1\right)\beta_i\gamma_0\sigma^2_\varepsilon
\end{aligned}
$$

and we can solve for the coefficients from:

$$
\begin{aligned}
0 &= \left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1-\rho\phi_3\right)\right)\phi_{0i}-\rho G\theta\left(\theta-1\right)K^{\theta-1}k\\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_2\right)^2\sigma^2_{\tilde{\varepsilon}}\\
&+ \frac{1}{2}\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_1\right)^2\beta_i^2\sigma^2_\varepsilon-\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_1\right)\beta_i\gamma_0\sigma^2_\varepsilon\\
&= \rho G\theta K^{\theta-1}\rho_z+\left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1+\rho\right)\right)\phi_2+\rho\xi\left(\rho_z+\phi_3\right)\phi_2\\
&= \rho G\theta K^{\theta-1}\rho_x+\left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1+\rho\right)\right)\phi_1+\rho\xi\left(\rho_x+\phi_3\right)\phi_1-\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_1\right)\gamma_1\sigma^2_\varepsilon\\
&= \left(\rho G\theta\left(\theta-1\right)K^{\theta-1}-\xi\left(1+\rho\right)\right)\phi_3+\rho\xi\phi_3^2+\xi
\end{aligned}
$$

Define $\hat{\xi}=\frac{\xi}{\rho G\theta K^{\theta-1}}=\frac{\xi}{1-\rho(1-\delta)}$. Then,

$$
\begin{aligned}
0 &= \Big(\left(\theta-1\right)-\hat{\xi}\left(1+\rho\right)\Big)\phi_3+\rho\hat{\xi}\phi_3^2+\hat{\xi}\\
\phi_1 &= \frac{\left(\rho_x-\gamma_1\sigma^2_\varepsilon\right)\phi_3}{\hat{\xi}\left(1-\rho\rho_x\phi_3+\rho\gamma_1\sigma^2_\varepsilon\phi_3\right)}\\
\phi_2 &= \frac{\rho_z\phi_3}{\hat{\xi}\left(1-\rho\rho_z\phi_3\right)}\\
\phi_{0i} &= \phi_{00}-\phi_{01}\beta_i+\phi_{02}\beta_i^2
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_{00} &= \frac{\rho G\theta\left(1-\theta\right)K^{\theta-1}k+\frac{1}{2}\left(\rho G\theta K^{\theta-1}+\rho\xi\phi_2\right)^2\sigma_{\tilde{\varepsilon}}^2}{\rho G\theta\left(1-\theta\right)K^{\theta-1}+\xi\left(1-\rho\phi_3\right)} \\
\phi_{01} &= \frac{\phi_3}{\hat{\xi}\left(1-\rho\phi_3\right)}\frac{\gamma_0\sigma_\varepsilon^2}{1-\rho\rho_x\phi_3+\rho\gamma_1\sigma_\varepsilon^2\phi_3} \\
\phi_{02} &= \frac{\rho G\theta K^{\theta-1}\rho\xi\phi_1+\frac{1}{2}\left(\rho\xi\phi_1\right)^2+\frac{1}{2}\left(\rho G\theta K^{\theta-1}\right)^2}{\rho G\theta\left(1-\theta\right)K^{\theta-1}+\xi\left(1-\rho\phi_3\right)}\sigma_\varepsilon^2
\end{aligned}
$$

Note that $\frac{\phi_3}{\hat{\xi}}$ goes to $\frac{1}{1-\theta}$ as $\hat{\xi}$ goes to zero and zero as $\hat{\xi}$ goes to infinity. Again ignoring variance terms, the policy function is

$$k_{it+1}=\phi_{00}+\phi_1\beta_i x_t+\phi_2 z_{it}+\phi_3 k_{it}-\phi_{01}\beta_i$$

which is equation (23) in the text.

MPK Dispersion. The expected mpk is given by

$$\mathbb{E}_t\left[mpk_{it+1}\right]=\log\theta+\log G+\beta_i\rho_x x_t+\rho_z z_{it}-\left(1-\theta\right)k_{it+1}$$

and the mean of this is

$$\mathbb{E}\left[\mathbb{E}_t\left[mpk_{it+1}\right]\right]=\log\theta+\log G-\left(1-\theta\right)\mathbb{E}\left[k_{it+1}\right]$$

From the policy function,

$$\mathbb{E}\left[k_{it+1}\right]=\frac{\phi_{00}-\phi_{01}\beta_i}{1-\phi_3}$$

so that

$$\mathbb{E}\left[\mathbb{E}_t\left[mpk_{it+1}\right]\right]=\log\theta+\log G-\frac{1-\theta}{1-\phi_3}\left(\phi_{00}-\phi_{01}\beta_i\right)$$

and the variance of this permanent component is

$$\sigma^2_{\mathbb{E}\left[\mathbb{E}_t\left[mpk_{it+1}\right]\right]}=\left(\frac{1-\theta}{1-\phi_3}\right)^2\phi_{01}^2\sigma_\beta^2$$

which is equation (24) in the text.

C.3 Aggregation

The first order condition on labor gives

$$N_{it} = \left(\frac{\theta_2 e^{\hat{\beta}_i x_t + \hat{z}_{it}} K_{it}^{\theta_1}}{W_t}\right)^{\frac{1}{1-\theta_2}}$$

and substituting for the wage,

$$N_{it} = \left(\theta_2 e^{\left(\hat{\beta}_i - \omega\right) x_t + \hat{z}_{it}} K_{it}^{\theta_1}\right)^{\frac{1}{1-\theta_2}}$$

Labor market clearing gives:

$$N_t = \int N_{it} di = \theta_2^{\frac{1}{1-\theta_2}} e^{-\frac{1}{1-\theta_2}\omega x_t} \int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di$$

so that

$$\theta_2^{\frac{\theta_2}{1-\theta_2}} e^{-\frac{\theta_2}{1-\theta_2}\omega x_t} = \left(\frac{N_t}{\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di}\right)^{\theta_2}$$

Then,

$$\begin{aligned} Y_{it} &= e^{\hat{\beta}_i x_t + \hat{z}_{it}} K_{it}^{\theta_1} N_{it}^{\theta_2} = \theta_2^{\frac{\theta_2}{1-\theta_2}} e^{-\frac{\theta_2}{1-\theta_2}\omega x_t} e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} \\ &= \frac{e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta}}{\left(\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di\right)^{\theta_2}} N_t^{\theta_2} \end{aligned}$$

By definition,

$$MPK_{it} = \frac{\theta e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta-1}}{\left(\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di\right)^{\theta_2}} N_t^{\theta_2}$$

and rearranging,

$$K_{it} = \left(\frac{\theta e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}}}{MPK_{it}}\right)^{\frac{1}{1-\theta}} \left(\frac{N_t}{\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di}\right)^{\frac{\theta_2}{1-\theta}}$$

Capital market clearing gives

$$K_t = \int K_{it} di = \theta^{\frac{1}{1-\theta}} \left(\frac{N_t}{\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t + z_{it}} K_{it}^{\theta} di}\right)^{\frac{\theta_2}{1-\theta}} \int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t + \frac{1}{1-\theta} z_{it}} MPK_{it}^{-\frac{1}{1-\theta}} di$$

so that

$$K_{it}^{\theta}=\left(\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}}{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}di}K_t\right)^{\theta}$$

and substituting into the expression for Y_{it},

$$\begin{aligned}
Y_{it} &= \frac{e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t+z_{it}}\left(\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}}{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}di}K_t\right)^{\theta}}{\left(\int e^{\frac{1}{1-\theta_2}\hat{\beta}_i x_t+z_{it}}\left(\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}}{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}di}K_t\right)^{\theta}di\right)^{\theta_2}}N_t^{\theta_2} \\
&= \frac{\frac{e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{\theta}{1-\theta}}}{\left(\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}di\right)^{\theta}}}{\left(\frac{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{\theta}{1-\theta}}di}{\left(\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}di\right)^{\theta}}\right)^{\theta_2}}K_t^{\theta_1}N_t^{\theta_2}
\end{aligned}$$

Aggregate output is then

$$Y_t=\int Y_{it}di=A_tK_t^{\theta_1}N_t^{\theta_2}$$

where

$$A_t=\left(\frac{\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{\theta}{1-\theta}}di}{\left(\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}di\right)^{\theta}}\right)^{1-\theta_2}$$

Taking logs,

$$a_t = (1-\theta_2)\left(\log\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{\theta}{1-\theta}}di-\theta\log\int e^{\frac{1}{1-\theta_2}\frac{1}{1-\theta}\hat{\beta}_i x_t+\frac{1}{1-\theta}z_{it}}MPK_{it}^{-\frac{1}{1-\theta}}di\right)$$

The first expression in braces is equal to

$$\begin{aligned}
&\frac{1}{1-\theta}\frac{1}{1-\theta_2}\bar{\hat{\beta}}x_t-\frac{\theta}{1-\theta}\bar{m}pk+\frac{1}{2}\left(\frac{1}{1-\theta}\right)^2\left(\left(\frac{1}{1-\theta_2}\right)^2x_t^2\sigma_{\hat{\beta}}^2+\sigma_z^2\right)+\frac{1}{2}\left(\frac{\theta}{1-\theta}\right)^2\sigma_{mpk}^2 \\
-&\frac{\theta}{(1-\theta)^2}\frac{1}{1-\theta_2}\sigma_{mpk,\hat{\beta}_i x_t+z_{it}}
\end{aligned}$$

and the second to

$$\frac{\theta}{1-\theta}\frac{1}{1-\theta_2}\bar{\hat{\beta}}x_t - \frac{\theta}{1-\theta}\bar{m}pk + \frac{1}{2}\theta\left(\frac{1}{1-\theta}\right)^2\left(\left(\frac{1}{1-\theta_2}\right)^2 x_t^2\sigma_{\hat{\beta}}^2 + \sigma_z^2\right) + \frac{1}{2}\theta\left(\frac{1}{1-\theta}\right)^2\sigma_{mpk}^2$$
$$- \quad \frac{\theta}{(1-\theta)^2}\frac{1}{1-\theta_2}\sigma_{mpk,\hat{\beta}_i x_t + z_{it}}$$

and combining (and using $\sigma_\beta = \frac{1}{1-\theta_2}\sigma_{\hat{\beta}}$) gives

$$\begin{aligned}
a_t &= \bar{\hat{\beta}}x_t + (1-\theta_2)\left(\frac{1}{2}\frac{1}{1-\theta}\left(x_t^2\sigma_\beta^2 + \sigma_z^2\right) - \frac{1}{2}\frac{\theta}{1-\theta}\sigma_{mpk}^2\right) \\
&= a_t^* - \frac{1}{2}(1-\theta_2)\frac{\theta}{1-\theta}\sigma_{mpk}^2 \\
&= a_t^* - \frac{1}{2}\frac{\theta_1(1-\theta_2)}{1-\theta_1-\theta_2}\sigma_{mpk}^2
\end{aligned}$$

C.4 Stock Market Returns

We derive stock market returns in the environment with adjustment costs. This nests the simpler case without them when $\xi = 0$.

Dividends are equal to

$$D_{it+1} = e^{z_{it+1}+\beta_i x_{t+1}}K_{it+1}^{\theta} - K_{it+2} + (1-\delta)K_{it+1} - \frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}} - 1\right)^2 K_{it+1}$$

and log-linearizing,

$$d_{it+1} = \frac{\Pi}{D}(z_{it+1} + \beta_i x_{t+1}) + \left(\theta\frac{\Pi}{D} + (1-\delta)\frac{K}{D}\right)k_{it+1} - \frac{K}{D}k_{it+2} + \log D - \left(\theta\frac{\Pi}{D} - \delta\frac{K}{D}\right)k$$

where $k = \log K$. Substituting for k_{it+1} and k_{it+2} from Appendix C.2.2 and rearranging,

$$d_{it+1} = A_{0i} + \tilde{A}_1 z_{it} + A_1\beta_i x_t + \tilde{A}_2\varepsilon_{it+1} + A_2\beta_i\varepsilon_{t+1} + A_3 k_{it}$$

where

$$\begin{aligned}
A_{0i} &= \log D - \left(\theta\frac{\Pi}{D} - \delta\frac{K}{D}\right)(k - \phi_{0i}) - \frac{K}{D}\phi_{0i}\phi_3 \\
A_1 &= \frac{\Pi}{D}\rho_x + \left(\theta\frac{\Pi}{D} + \frac{K}{D}(1-\delta-\rho_x-\phi_3)\right)\phi_1 \\
\tilde{A}_1 &= \frac{\Pi}{D}\rho_z + \left(\theta\frac{\Pi}{D} + \frac{K}{D}(1-\delta-\rho_z-\phi_3)\right)\phi_2 \\
A_2 &= \frac{\Pi}{D} - \frac{K}{D}\phi_1 \\
\tilde{A}_2 &= \frac{\Pi}{D} - \frac{K}{D}\phi_2 \\
A_3 &= \left(\theta\frac{\Pi}{D} + \frac{K}{D}(1-\delta-\phi_3)\right)\phi_3
\end{aligned}$$

By definition, returns are equal to

$$R_{it+1} = \frac{D_{it+1} + P_{it+1}}{P_{it}}$$

and log-linearizing,

$$r_{it+1} = \rho p_{it+1} + (1-\rho)\, d_{it+1} - p_{it} - \log\rho + (1-\rho)\log\frac{P}{D}$$

Conjecture the stock price takes the form

$$p_{it} = c_{0i} + c_1\beta_i x_t + c_2 z_{it} + c_3 k_{it}$$

Then,

$$\begin{aligned}
r_{it+1} &= -\log\rho + (1-\rho)\left(\log\frac{P}{D} + A_{0i} - c_{0i}\right) + \rho c_3\phi_{0i} \\
&+ \left((\rho\rho_z - 1)\, c_2 + \rho c_3\phi_2 + (1-\rho)\,\tilde{A}_1\right) z_{it} \\
&+ \left((\rho\rho_x - 1)\, c_1 + \rho c_3\phi_1 + (1-\rho)\, A_1\right)\beta_i x_t \\
&+ \left((\rho\phi_3 - 1)\, c_3 + (1-\rho)\, A_3\right) k_{it} \\
&+ \left(\rho c_2 + (1-\rho)\,\tilde{A}_2\right)\varepsilon_{it+1} + \left(\rho c_1 + (1-\rho)\, A_2\right)\beta_i\varepsilon_{t+1}
\end{aligned}$$

and the (log) excess return is the (negative of the) conditional covariance with the SDF:

$$\log\mathbb{E}_t\left[R^e_{it+1}\right] = \left(\rho c_1 + (1-\rho)\, A_2\right)\beta_i\gamma_t\sigma^2_\varepsilon$$

To solve for coefficients, use the Euler equation. First,

$$
\begin{aligned}
r_{it+1}+m_{it+1} &= (1-\rho)\left(\log\frac{P}{D}+A_{0i}-c_{0i}\right)+\rho c_3\phi_{0i}-\frac{1}{2}\gamma_0^2\sigma_\varepsilon^2 \\
&+ \left((\rho\rho_z-1)\,c_2+\rho c_3\phi_2+(1-\rho)\,\tilde{A}_1\right)z_{it} \\
&+ \left(((\rho\rho_x-1)\,c_1+\rho c_3\phi_1+(1-\rho)\,A_1)\,\beta_i-\gamma_0\gamma_1\sigma_\varepsilon^2\right)x_t \\
&+ ((\rho\phi_3-1)\,c_3+(1-\rho)\,A_3)\,k_{it} \\
&- \frac{1}{2}\gamma_1^2\sigma_\varepsilon^2x_t^2 \\
&+ \left(\rho c_2+(1-\rho)\,\tilde{A}_2\right)\varepsilon_{it+1} \\
&+ ((\rho c_1+(1-\rho)\,A_2)\,\beta_i-\gamma_0-\gamma_1x_t)\,\varepsilon_{t+1}
\end{aligned}
$$

The Euler equation implies

$$
\begin{aligned}
0 &= \mathbb{E}_t\left[r_{it+1}+m_{it+1}\right]+\frac{1}{2}\mathrm{var}\left(r_{it+1}+m_{it+1}\right) \\
&= (1-\rho)\left(\log\frac{P}{D}+A_{0i}-c_{0i}\right)+\rho c_3\phi_{0i}+\frac{1}{2}\left(\rho c_1+(1-\rho)\,A_2\right)^2\beta_i^2\sigma_\varepsilon^2-\left(\rho c_1+(1-\rho)\,A_2\right)\beta_i\gamma_0\sigma_\varepsilon^2 \\
&+ \frac{1}{2}\left(\rho c_2+(1-\rho)\,\tilde{A}_2\right)^2\sigma_{\tilde{\varepsilon}}^2 \\
&+ \left((\rho\rho_z-1)\,c_2+\rho c_3\phi_2+(1-\rho)\,\tilde{A}_1\right)z_{it} \\
&+ \left((\rho\rho_x-1)\,c_1+\rho c_3\phi_1+(1-\rho)\,A_1-(\rho c_1+(1-\rho)\,A_2)\,\gamma_1\sigma_\varepsilon^2\right)\beta_ix_t \\
&+ ((\rho\phi_3-1)\,c_3+(1-\rho)\,A_3)\,k_{it}
\end{aligned}
$$

and so by undetermined coefficients,

$$
\begin{aligned}
0 &= (1-\rho)\left(\log\frac{P}{D}+A_{0i}-c_{0i}\right)+\rho c_3\phi_{0i}+\frac{1}{2}\left(\rho c_1+(1-\rho)\,A_2\right)^2\beta_i^2\sigma_\varepsilon^2-\left(\rho c_1+(1-\rho)\,A_2\right)\beta_i\gamma_0\sigma_\varepsilon^2 \\
&+ \frac{1}{2}\left(\rho c_2+(1-\rho)\,\tilde{A}_2\right)^2\sigma_{\tilde{\varepsilon}}^2 \\
&= (\rho\rho_z-1)\,c_2+\rho c_3\phi_2+(1-\rho)\,\tilde{A}_1 \\
&= (\rho\rho_x-1)\,c_1+\rho c_3\phi_1+(1-\rho)\,A_1-(\rho c_1+(1-\rho)\,A_2)\,\gamma_1\sigma_\varepsilon^2 \\
&= (\rho\phi_3-1)\,c_3+(1-\rho)\,A_3
\end{aligned}
$$

or

$$
\begin{aligned}
c_3 &= \frac{(1-\rho)\,A_3}{1-\rho\phi_3} \\
c_2 &= \frac{\rho c_3\phi_2 + (1-\rho)\,\tilde{A}_1}{1-\rho\rho_z} \\
c_1 &= \frac{\rho c_3\phi_1 + (1-\rho)\left(A_1 - A_2\gamma_1\sigma_\varepsilon^2\right)}{1-\rho\rho_x+\rho\gamma_1\sigma_\varepsilon^2}
\end{aligned}
$$

Substituting for c_1 we can solve for

$$
\log \mathbb{E}_t\left[R^e_{it+1}\right] = \frac{\rho^2 c_3\phi_1 + (1-\rho)\left(\rho A_1 + (1-\rho\rho_x)\,A_2\right)}{1-\rho\rho_x+\rho\sigma_\varepsilon^2\gamma_1}\beta_i\gamma_t\sigma_\varepsilon^2
$$

Solving for

$$
\begin{aligned}
\rho A_1 + (1-\rho\rho_x)\,A_2 &= \frac{\frac{1}{\rho}+\delta-1-\rho\theta\phi_1\phi_3}{\frac{1}{\rho}+\delta\,(1-\theta)-1} \\
\rho^2 c_3\phi_1 &= \theta\frac{\rho^2\,(1-\rho)\,\phi_1\phi_3}{1-\rho\phi_3}\frac{\frac{1}{\rho}-\phi_3}{\frac{1}{\rho}+\delta\,(1-\theta)-1}
\end{aligned}
$$

substituting into the return equation and simplifying, we obtain

$$
\log \mathbb{E}_t\left[R^e_{it+1}\right] = \psi\beta_i\gamma_t\sigma_\varepsilon^2
$$

where

$$
\psi = \frac{\frac{1}{\rho}+\delta-1}{\frac{1}{\rho}+\delta\,(1-\theta)-1}\frac{1-\rho}{1-\rho\rho_x+\rho\gamma_1\sigma_\varepsilon^2}
$$

which is equation (19) in the text.

The Sharpe ratio is the ratio of expected excess returns to the conditional standard deviation of the return:

$$
SR_{it} = \frac{\psi\beta_i\gamma_t\sigma_\varepsilon^2}{\sqrt{\left(\rho c_2 + (1-\rho)\,\tilde{A}_2\right)^2\sigma_{\tilde{\varepsilon}}^2 + \psi^2\beta_i^2\sigma_\varepsilon^2}}
$$

We can solve for

$$
\rho c_2 + (1-\rho)\,\tilde{A}_2 = \frac{\frac{1}{\rho}+\delta-1}{\frac{1}{\rho}+\delta\,(1-\theta)-1}\frac{1-\rho}{1-\rho\rho_z}
$$

and substituting and rearranging gives the expression in footnote 36.

For a perfectly diversified portfolio (i.e., the integral over individual returns) idiosyncratic shocks cancel, i.e., $\sigma_{\tilde{\varepsilon}}^2 = 0$ and $SR_{mt} = \gamma_t\sigma_\varepsilon$.

C.5 Autocorrelation of Investment

To derive the autocorrelation of investment, define net investment as $\Delta k_{it+1} = k_{it+1} - k_{it}$. We use the following:

$$\begin{aligned}
\mathrm{cov}\left(\Delta z_{it}, z_{it}\right) &= \mathrm{cov}\left(\left(\rho_z - 1\right) z_{it-1} + \varepsilon_{it}.\rho_z z_{it-1} + \varepsilon_{it}\right) \\
&= \rho_z \left(\rho_z - 1\right) \sigma_z^2 + \sigma_{\tilde{\varepsilon}}^2 \\
&= \frac{1}{1+\rho_z} \sigma_{\tilde{\varepsilon}}^2 \\
\mathrm{cov}\left(\Delta k_{it}, z_{it}\right) &= \mathrm{cov}\left(\Delta k_{it}, \rho_z z_{it-1} + \varepsilon_{it}\right) \\
&= \rho_z \mathrm{cov}\left(\Delta k_{it}, z_{it-1}\right) \\
&= \rho_z \mathrm{cov}\left(\phi_1 \beta_i \Delta x_{t-1} + \phi_2 \Delta z_{it-1} + \phi_3 \Delta k_{it-1}, z_{it-1}\right) \\
&= \rho_z \left(\mathrm{cov}\left(\phi_2 \Delta z_{it-1}, z_{it-1}\right) + \phi_3 \mathrm{cov}\left(\Delta k_{it-1}, z_{it-1}\right)\right) \\
&= \rho_z \phi_2 \frac{1}{1+\rho_z} \sigma_{\tilde{\varepsilon}}^2 + \rho_z \phi_3 \mathrm{cov}\left(\Delta k_{it-1}, z_{it-1}\right)
\end{aligned}$$

so that

$$\mathbb{E}\left[\mathrm{cov}\left(\Delta k_{it}, z_{it}\right)\right] = \frac{\rho_z}{1+\rho_z} \frac{\phi_2 \sigma_{\tilde{\varepsilon}}^2}{1 - \phi_3 \rho_z}$$

Next,

$$\begin{aligned}
\mathrm{cov}\left(\Delta k_{it+1}, \Delta z_{it+1}\right) &= \mathrm{cov}\left(\phi_1 \beta_i \Delta x_t + \phi_2 \Delta z_{it} + \phi_3 \Delta k_{it}.\left(\rho_z - 1\right) z_{it} + \varepsilon_{it+1}\right) \\
&= \phi_2 \left(\rho_z - 1\right) \mathrm{cov}\left(\Delta z_{it}, z_{it}\right) + \phi_3 \left(\rho_z - 1\right) \mathrm{cov}\left(\Delta k_{it}, z_{it}\right) \\
&= \phi_2 \left(\rho_z - 1\right) \frac{1}{1+\rho_z} \sigma_{\tilde{\varepsilon}}^2 + \frac{\rho_z}{1+\rho_z} \frac{\phi_3 \left(\rho_z - 1\right) \phi_2 \sigma_{\tilde{\varepsilon}}^2}{1 - \phi_3 \rho_z} \\
&= \frac{\rho_z - 1}{1+\rho_z} \frac{\phi_2 \sigma_{\tilde{\varepsilon}}^2}{1 - \phi_3 \rho_z}
\end{aligned}$$

Similar steps give

$$\mathrm{cov}\left(\Delta k_{it+1}, \Delta x_{t+1}\right) = \frac{\rho_x - 1}{1+\rho_x} \frac{\phi_1 \beta_i \sigma_{\varepsilon}^2}{1 - \phi_3 \rho_x}$$

Combining these gives the variance of investment:

$$\begin{aligned}
\sigma^2_{\Delta k} &= \phi_1^2\beta_i^2 \text{var}\left(\Delta x_t\right) + \phi_2^2 \text{var}\left(\Delta z_{it}\right) + \phi_3^2\sigma^2_{\Delta k} \\
&+ 2\phi_1\phi_3\beta_i \text{cov}\left(\Delta x_t, \Delta k_{it}\right) + 2\phi_2\phi_3 \text{cov}\left(\Delta z_{it}, \Delta k_{it}\right) \\
&= \phi_1^2\beta_i^2 \frac{2}{1+\rho_x}\sigma^2_\varepsilon + \phi_2^2 \frac{2}{1+\rho_z}\sigma^2_{\tilde{\varepsilon}} + \phi_3^2\sigma^2_{\Delta k} \\
&+ \frac{2\phi_1^2\phi_3\beta_i^2\sigma^2_\varepsilon}{1-\phi_3\rho_x}\frac{\rho_x-1}{1+\rho_x} + \frac{2\phi_2^2\phi_3\sigma^2_{\tilde{\varepsilon}}}{1-\phi_3\rho_z}\frac{\rho_z-1}{1+\rho_z} \\
&= \phi_3^2\sigma^2_{\Delta k} + 2\phi_1^2\beta_i^2\frac{\sigma^2_\varepsilon}{1+\rho_x}\left(1+\frac{\phi_3\left(\rho_x-1\right)}{1-\phi_3\rho_x}\right) + 2\phi_2^2\frac{\sigma^2_{\tilde{\varepsilon}}}{1+\rho_z}\left(1+\frac{\phi_3\left(\rho_z-1\right)}{1-\phi_3\rho_z}\right) \\
&= \frac{2}{1+\phi_3}\left(\phi_1^2\beta_i^2\sigma^2_\varepsilon\frac{1}{1+\rho_x}\frac{1}{1-\phi_3\rho_x} + \phi_2^2\sigma^2_{\tilde{\varepsilon}}\frac{1}{1+\rho_z}\frac{1}{1-\phi_3\rho_z}\right)
\end{aligned}$$

Next,

$$\begin{aligned}
\text{cov}\left(\Delta k_{it+1}, \Delta k_{it}\right) &= \text{cov}\left(\phi_1\beta_i\Delta x_t + \phi_2\Delta z_{it} + \phi_3\Delta k_{it}, \Delta k_{it}\right) \\
&= \phi_1\beta_i \text{cov}\left(\Delta x_t, \Delta k_{it}\right) + \phi_2 \text{cov}\left(\Delta z_{it}, \Delta k_{it}\right) + \phi_3\sigma^2_{\Delta k} \\
&= \phi_1^2\beta_i^2\sigma^2_\varepsilon\frac{\rho_x-1}{1+\rho_x}\frac{1}{1-\phi_3\rho_x} + \phi_2^2\sigma^2_{\tilde{\varepsilon}}\frac{\rho_z-1}{1+\rho_z}\frac{1}{1-\phi_3\rho_z} + \phi_3\sigma^2_{\Delta k}
\end{aligned}$$

and the autocorrelation is:

$$\text{corr}\left(\Delta k_{it+1}, \Delta k_{it}\right) = \phi_3 + \frac{1+\phi_3}{2}\frac{\phi_1^2\beta_i^2\sigma^2_\varepsilon\frac{\rho_x-1}{1+\rho_x}\frac{1}{1-\phi_3\rho_x} + \phi_2^2\sigma^2_{\tilde{\varepsilon}}\frac{\rho_z-1}{1+\rho_z}\frac{1}{1-\phi_3\rho_z}}{\phi_1^2\beta_i^2\sigma^2_\varepsilon\frac{1}{1+\rho_x}\frac{1}{1-\phi_3\rho_x} + \phi_2^2\sigma^2_{\tilde{\varepsilon}}\frac{1}{1+\rho_z}\frac{1}{1-\phi_3\rho_z}} \tag{32}$$

Notice that this approaches

$$\text{corr}\left(\Delta k_{it+1}, \Delta k_{it}\right) = \phi_3 + \left(1-\phi_3\right)\frac{\rho_x-1}{2}$$

as ρ_z and ρ_x become close. Further, in the case both shocks follow a random walk, the autocorrelation is simply equal to ϕ_3.

C.6 Other Distortions

With other distortions, the derivations are similar to those in Appendix C.2.1. The Euler equation is given by

$$\begin{aligned}
1 &= \mathbb{E}_t\left[M_{t+1}\left(\theta e^{\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}} G K_{it+1}^{\theta-1} + 1 - \delta\right)\right] \\
&= \left(1-\delta\right)\mathbb{E}_t\left[M_{t+1}\right] + \theta G K_{it+1}^{\theta-1}\mathbb{E}_t\left[e^{m_{t+1}+\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}}\right]
\end{aligned}$$

Idiosyncratic distortions. Substituting for m_{t+1} and τ_{it+1} and rearranging,

$$\begin{aligned}
\mathbb{E}_t\left[e^{m_{t+1}+\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}}\right] &= \mathbb{E}_t\left[e^{\log\rho-\gamma_t\varepsilon_{t+1}-\frac{1}{2}\gamma_t^2\sigma_\varepsilon^2-\nu z_{it+1}-\eta_{it+1}+z_{it+1}+\beta_i x_{t+1}}\right]\\
&= \mathbb{E}_t\left[e^{\log\rho+(1-\nu)\rho_z z_{it}+(1-\nu)\varepsilon_{it+1}+\beta_i\rho_x x_t+(\beta_i-\gamma_t)\varepsilon_{t+1}-\frac{1}{2}\gamma_t^2\sigma_\varepsilon^2-\eta_{it+1}}\right]\\
&= e^{\log\rho+(1-\nu)\rho_z z_{it}+\beta_i\rho_x x_t+\frac{1}{2}(1-\nu)^2\sigma_{\tilde{\varepsilon}}^2+\frac{1}{2}\beta_i^2\sigma_\varepsilon^2-\beta_i\gamma_t\sigma_\varepsilon^2-\eta_{it+1}}
\end{aligned}$$

so that

$$\theta G K_{it+1}^{\theta-1} = \frac{1-(1-\delta)\rho}{e^{\log\rho+(1-\nu)\rho_z z_{it}+\beta_i\rho_x x_t+\frac{1}{2}(1-\nu)^2\sigma_{\tilde{\varepsilon}}^2+\frac{1}{2}\beta_i^2\sigma_\varepsilon^2-\beta_i\gamma_t\sigma_\varepsilon^2-\eta_{it+1}}}$$

and rearranging and taking logs,

$$k_{it+1} = \frac{1}{1-\theta}\left(\tilde{\alpha}+\frac{1}{2}(1-\nu)^2\sigma_{\tilde{\varepsilon}}^2+\frac{1}{2}\beta_i^2\sigma_\varepsilon^2+(1-\nu)\rho_z z_{it}+\beta_i\rho_x x_t-\beta_i\gamma_t\sigma_\varepsilon^2-\eta_{it+1}\right)$$

where $\tilde{\alpha}$ and α are as defined in Appendix C.2.1.

The realized mpk is given by (ignoring the variance terms)

$$\begin{aligned}
mpk_{it+1} &= \log\theta+\pi_{it+1}-k_{it+1}\\
&= \log\theta+\log G+z_{it+1}+\beta_i x_{t+1}-(1-\theta)k_{it+1}\\
&= \log\theta+\log G+z_{it+1}+\beta_i x_{t+1}-\tilde{\alpha}-(1-\nu)\rho_z z_{it}-\beta_i\rho_x x_t+\beta_i\gamma_t\sigma_\varepsilon^2+\eta_{it+1}\\
&= \alpha+\varepsilon_{it+1}+\beta_i\varepsilon_{t+1}+\nu\rho_z z_{it}+\beta_i\gamma_t\sigma_\varepsilon^2+\eta_{it+1}
\end{aligned}$$

which is equation (26). The conditional expected mpk is

$$\mathbb{E}_t\left[mpk_{it+1}\right] = \alpha+\nu\rho_z z_{it}+\beta_i\gamma_t\sigma_\varepsilon^2+\eta_{it+1}$$

and the cross-sectional variance is

$$\sigma^2_{\mathbb{E}_t[mpk_{it+1}]} = (\nu\rho_z)^2\sigma_z^2+\sigma_\eta^2+\left(\gamma_t\sigma_\varepsilon^2\right)^2\sigma_\beta^2 \tag{33}$$

Deriving stock returns follows closely the steps in Appendix C.4. Dividends are equal to

$$D_{it+1} = e^{\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}}K_{it+1}^\theta-K_{it+2}+(1-\delta)K_{it+1}-\frac{\xi}{2}\left(\frac{K_{it+2}}{K_{it+1}}-1\right)^2 K_{it+1}$$

and log-linearizing,

$$d_{it+1} = \frac{\Pi}{D}\left(\tau_{it+1}+z_{it+1}+\beta_i x_{t+1}\right)+\left(\theta\frac{\Pi}{D}+(1-\delta)\frac{K}{D}\right)k_{it+1}-\frac{K}{D}k_{it+2}+\log D-\left(\theta\frac{\Pi}{D}-\delta\frac{K}{D}\right)k$$

where $k = \log K$.

Substituting for k_{it+1} and k_{it+2} from above,

$$d_{it+1} = A_0 + \tilde{A}_1 z_{it} + A_1 \beta_i x_t + \tilde{A}_2 \varepsilon_{it+1} + A_2 \beta_i \varepsilon_{t+1} + A_3 \eta_{it+1} + A_4 \eta_{it+2}$$

where

$$\begin{aligned}
A_0 &= \log D - \left(\theta \frac{\Pi}{D} - \delta \frac{K}{D}\right)\left(k - \frac{\tilde{\alpha}}{1-\theta}\right) \\
A_1 &= \frac{1}{1-\theta}\left(\frac{\Pi}{D} + (1-\delta-\rho_x)\frac{K}{D}\right)\rho_x - \frac{1}{1-\theta}\left(\theta\frac{\Pi}{D} + (1-\delta-\rho_x)\frac{K}{D}\right)\gamma_1 \sigma_\varepsilon^2 \\
\tilde{A}_1 &= \frac{1-\nu}{1-\theta}\left(\frac{\Pi}{D} + (1-\delta-\rho_z)\frac{K}{D}\right)\rho_z \\
A_2 &= \frac{\Pi}{D} - \frac{1}{1-\theta}\frac{K}{D}\rho_x + \frac{1}{1-\theta}\frac{K}{D}\gamma_1 \sigma_\varepsilon^2 \\
\tilde{A}_2 &= \left(\frac{\Pi}{D} - \frac{1}{1-\theta}\frac{K}{D}\right)(1-\nu)\rho_z \\
A_3 &= -\frac{1}{1-\theta}\left(\theta\frac{\Pi}{D} + (1-\delta)\frac{K}{D}\right) \\
A_4 &= \frac{1}{1-\theta}\frac{K}{D}
\end{aligned}$$

Using the log-linearized return equation,

$$r_{it+1} = \rho p_{it+1} + (1-\rho) d_{it+1} - p_{it} - \log\rho + (1-\rho)\log\frac{P}{D}$$

and conjecturing the stock price takes the form

$$p_{it} = c_{0i} + c_1 \beta_i x_t + c_2 z_{it} + c_3 \eta_{it+1}$$

gives

$$\begin{aligned}
r_{it+1} &= -\log\rho + (1-\rho)\left(\log\frac{P}{D} + A_0 - c_0\right) \\
&+ \left((\rho\rho_z - 1)c_2 + (1-\rho)\tilde{A}_1\right) z_{it} \\
&+ ((\rho\rho_x - 1)c_1 + (1-\rho)A_1)\beta_i x_t \\
&+ \left(\rho c_2 + (1-\rho)\tilde{A}_2\right)\varepsilon_{it+1} + (\rho c_1 + (1-\rho)A_2)\beta_i \varepsilon_{t+1} \\
&+ (\rho c_3 + (1-\rho)A_4)\eta_{it+2} + ((1-\rho)A_3 - c_3)\eta_{it+1}
\end{aligned}$$

The (log) excess return is the (negative of the) conditional covariance with the SDF:

$$\log \mathbb{E}_t \left[R^e_{it+1}\right] = \left(\rho c_1 + (1-\rho) A_2\right) \beta_i \gamma_t \sigma^2_\varepsilon$$

A_2 is independent of ν and η. Following the same steps as in Appendix C.4, it is easily verified that c_1 is independent of these terms as well. Thus, expected returns are independent of distortions.

Aggregate distortions. Consider the first formulation, i.e.,

$$\tau_{it+1} = -\nu_z z_{it+1} - \nu_x x_{t+1} - \eta_{it+1}$$

Similar steps as above give expression (33). Dispersion in expected stock market returns are similarly unaffected.

Next, consider the second formulation:

$$\tau_{it+1} = -\nu_z z_{it+1} - \nu_x \beta_i x_{t+1} - \eta_{it+1}$$

In this case, similar steps as above give the conditional expected *mpk* as

$$\mathbb{E}_t \left[mpk_{it+1}\right] = \alpha + \nu_z \rho_z z_{it} + \nu_x \beta_i \rho_x x_t + (1-\nu_x) \beta_i \gamma_t \sigma^2_\varepsilon + \eta_{it+1}$$

and expected excess stock market returns as

$$\log \mathbb{E}_t \left[R^e_{it+1}\right] = (1-\nu_x) \psi \beta_i \gamma_t \sigma^2_\varepsilon$$

where ψ is as defined in expression (19). In other words, the risk-premium effect on expected *mpk*, as well as expected returns, are both scaled by a factor $1-\nu_x$.

The mean level of expected *mpk* and return dispersion are, respectively,

$$\begin{aligned}
\mathbb{E}\left[\sigma^2_{\mathbb{E}_t[mpk_{it+1}]}\right] &= \sigma^2_\eta + (\nu_z \rho_z)^2 \sigma^2_z + (\nu_x \rho_x)^2 \sigma^2_x \sigma^2_\beta \\
&+ \left((1-\nu_x)\sigma^2_\varepsilon\right)^2 \left(\gamma_0^2 + \gamma_1^2 \sigma^2_x\right) \sigma^2_\beta + 2\nu_x (1-\nu_x) \rho_x \sigma^2_x \gamma_1 \sigma^2_\varepsilon \sigma^2_\beta \\
\mathbb{E}\left[\sigma^2_{\log \mathbb{E}_t\left[R^e_{it+1}\right]}\right] &= \left((1-\nu_x)\psi\sigma^2_\varepsilon\right)^2 \left(\gamma_0^2 + \gamma_1^2 \sigma^2_x\right) \sigma^2_\beta
\end{aligned}$$

The last two terms of the first equation capture the *mpk* effects of risk premia. The last term there is new and does not have a counterpart in the second equation – in other words, using dispersion in expected returns would give the second to last term, as usual, but not the last. If $\nu_x < 0$, it is straightforward to verify that that term is positive (recall that γ_1 is negative).

Then, we may be understating risk premium effects. If $\nu_x > 0$, the last terms is negative and we may be overstating them. At the estimated parameter values, the upper bound on ν_x discussed in Section 5.3 is about 0.065. Using this value, along with the other parameters, to calculate the last term in the equation gives the maximum upward bias to be 0.006, which is extremely small relative to our estimates of dispersion in *Empk* arising from the risk premium channel.

C.7 Multifactor Model

There are J aggregate risk factors in the economy. Firms have heterogeneous loadings on these factors, so that the profit function (in logs) takes the form

$$\pi_{it} = \beta_i x_t + z_{it} + \theta k_{it} \tag{34}$$

where β_i is a vector of factor loadings of firm i and x_t the vector of factor realizations at time t, i.e.,

$$\beta_i = \begin{bmatrix} \beta_{1i} \\ \beta_{2i} \\ \vdots \\ \beta_{Ji} \end{bmatrix}' \qquad x_t = \begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{Jt} \end{bmatrix}'$$

Each factor, indexed by j, follows an AR(1) process

$$x_{jt+1} = \rho_j x_{jt} + \varepsilon_{jt+1}, \quad \varepsilon_{jt+1} \sim \mathcal{N}\left(0, \sigma^2_{\varepsilon_j}\right) \tag{35}$$

where the innovations are potentially correlated across factors. Denote by Σ_f the covariance matrix of factor innovations, i.e.,

$$\Sigma_f = \begin{bmatrix} \sigma^2_{\varepsilon_1} & \sigma_{\varepsilon_1,\varepsilon_2} & \cdots & \sigma_{\varepsilon_1,\varepsilon_J} \\ \sigma_{\varepsilon_2,\varepsilon_1} & \sigma^2_{\varepsilon_2} & \cdots & \sigma_{\varepsilon_2,\varepsilon_J} \\ \vdots & \vdots & & \vdots \\ \sigma_{\varepsilon_J,\varepsilon_1} & \sigma_{\varepsilon_J,\varepsilon_2} & \cdots & \sigma^2_{\varepsilon_J} \end{bmatrix}$$

The idiosyncratic component of firm productivity follows

$$z_{it+1} = \rho_z z_{it} + \varepsilon_{it+1}, \quad \varepsilon_{it+1} \sim \mathcal{N}\left(0, \sigma^2_{\tilde{\varepsilon}}\right) \tag{36}$$

The stochastic discount factor takes the form

$$m_{t+1} = \log \rho - \gamma \varepsilon_{t+1} - \frac{1}{2}\gamma \Sigma_f \gamma' \tag{37}$$

where γ is a vector of factor exposures and ε_{t+1} the vector of innovations in each factor, i.e.,

$$\gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_J \end{bmatrix}' \qquad \varepsilon_{t+1} = \begin{bmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \\ \vdots \\ \varepsilon_{Jt+1} \end{bmatrix}$$

For simplicity, we have assumed that the factor exposures are constant, although the setup can be extended to include time-varying exposures as well. Expressions (34), (35), (36) and (37) are simple extensions of (9), (7) and (8).

Following a similar derivation as C.2.1, we can derive the realized *mpk*:

$$mpk_{it+1} = \alpha + \varepsilon_{it+1} + \beta_i \varepsilon_{t+1} + \beta_i \Sigma_f \gamma'$$

where β_i and ε_{t+1} denote vectors of factor loadings and shocks. The expected *mpk* and its cross-sectional dispersion are given by

$$\mathbb{E}_t \left[mpk_{it+1} \right] = \alpha + \beta_i \Sigma_f \gamma', \quad \sigma^2_{\mathbb{E}_t[mpk]} = \gamma \Sigma_f' \Sigma_\beta \Sigma_f \gamma'$$

where Σ_β is the covariance matrix of factor loadings across firms, i.e.,

$$\Sigma_\beta = \begin{bmatrix} \sigma^2_{\beta_1} & \sigma_{\beta_1,\beta_2} & \cdots & \sigma_{\beta_1,\beta_J} \\ \sigma_{\beta_2,\beta_1} & \sigma^2_{\beta_2} & \cdots & \sigma_{\beta_2,\beta_J} \\ \vdots & \vdots & & \vdots \\ \sigma_{\beta_J,\beta_1} & \sigma_{\beta_J,\beta_2} & \cdots & \sigma^2_{\beta_J} \end{bmatrix}$$

Similar steps as Appendix C.4 gives

$$\mathbb{E}_t \left[R^e_{it+1} \right] = \beta_i \psi \Sigma_f \gamma', \quad \sigma^2_{\mathbb{E}_t[R]} = \gamma \Sigma_f' \psi' \Sigma_\beta \psi \Sigma_f \gamma'$$

where ψ is a diagonal matrix with

$$\psi_{jj} = \left(\frac{1-\rho}{1-\rho\rho_j} \right) \frac{\frac{1}{\rho} + \delta - 1}{\frac{1}{\rho} + (1-\theta)\,\delta - 1}$$

D Additional Portfolio Sorts

This appendix reports additional portfolio sorts and summary statistics by portfolio.

Portfolio summary statistics. Table 11 displays summary statistics of firm characteristics across the industry-adjusted MPK-sorted portfolios.[56] A few observations are in order: while size and book-to-market seem to be correlated with firm MPK, the sorting is not monotonic. There are not large differences in the leverage of high and low MPK firms. One possible concern is that our measure of capital omits intangible capital, and that firms that seem to have high MPK (low capital capital utilization) are using intangible capital instead of physical capital. The table shows that this is unlikely to be the case – firms with low MPK, who use capital more intensively, also use intangible capital more intensively, as shown by their relatively high research and development (R&D) expenditures (relative to sales) and also their relatively high sales, general, and administrative (SG&A) expenses (relative to sales), two commonly used measures of investment in intangible capital.

Table 11: Firm Characteristics Across MPK-Sorted Portfolios

Portfolio	Low	2	3	4	High	Pooled
mpk	0.588	1.254	1.572	1.987	2.793	1.639
Market Capitalization	202.8	344.4	355.9	206.2	97.42	241.4
Sales	111.4	333.1	372.2	239.6	104.5	232.2
PPENT	55.31	103.2	87.84	33.25	7.012	57.33
Book Assets	189.6	374.6	385.7	202.3	85.24	247.5
Book to Market Ratio	0.636	0.758	0.765	0.708	0.622	0.698
Profitability	0.069	0.120	0.125	0.123	0.102	0.108
Market Leverage	0.242	0.300	0.307	0.289	0.267	0.281
R&D to Sales Ratio	0.112	0.051	0.044	0.052	0.065	0.064
SGA to Sales Ratio	0.317	0.251	0.239	0.260	0.283	0.270

Notes: This table reports the characteristics of firms sorted into five portfolios based on their industry-adjusted *mpk*. Firms are formed into portfolios annually by their (de-meaned by industry-year) *mpk*, for those industry-years with at least 10 firms. We compute the median value for each characteristic for each portfolio in each year, and then average those portfolio medians over time. The stock variables (market capitalization, sales, ppent, book assets) are in millions of 2009 dollars, deflated by the annual CPI. All other variables are ratios. R&D is research and development expenses from Compustat, while SGA is sales, general, and administrative expenses, a measure often associated with intangible capital. Further details on our computation of these measures can be found in appendix A.

Portfolio sorts - robustness. Table 12 reports two additional measures of excess returns across portfolios. The first, r^e_{t+3} computes three year ahead excess returns (compared to one-year ahead in Table 1). The second, r^a_{t+1} computes one year ahead unlevered returns, which we calculate using an unlimited liability model. The differences in high versus low MPK portfolio returns are robust to these alternatives (for example, the return to the MPK-HML portfolio continues to be both economically and statistically significant, ranging from about 2% to 3.5%).

[56]The table displays median firm characteristics, but the means yield qualitatively similar patterns.

Table 12: Excess Returns on MPK-Sorted Portfolios – Robustness

Portfolio	Low	1	2	3	4	MPK-HML
			Panel A: Not Industry-Adjusted			
r^e_{t+3}	9.63***	12.43***	12.69***	13.90***	12.99***	3.36*
	(2.96)	(3.69)	(3.71)	(3.81)	(3.38)	(1.96)
r^a_{t+1}	4.64*	7.53***	8.69***	8.66***	8.22***	3.58***
	(1.88)	(3.07)	(3.53)	(3.26)	(3.02)	(3.05)
			Panel B: Industry-Adjusted			
r^e_{t+3}	11.95***	12.27***	12.04***	12.60***	13.82***	1.87**
	(2.99)	(3.71)	(3.75)	(3.60)	(3.58)	(2.22)
r^a_{t+1}	6.86**	7.16***	8.04***	8.17***	8.84***	1.97***
	(2.13)	(2.94)	(3.37)	(3.15)	(3.04)	(2.66)

Notes: This table reports stock market returns for portfolios sorted by *mpk*. r^e_{t+3} denotes equal-weighted annualized monthly excess stock returns (over the risk-free rate) measured from July of year $t+3$ to June of year $t+4$. r^a_{t+1} denotes equal-weighted unlevered ("asset") returns from from July of year $t+1$ to June of year $t+2$, where we use an unlimited liability model to unlever equity returns. Industry adjustment is done by demeaning *mpk* by industry-year and sorting portfolios on de-meaned *mpk*, where industries are defined at the 4-digit SIC code level. t-statistics in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 13 reports the results of the portfolio sorts across 10, rather than 5, portfolios. We report contemporaneous returns, r^e_t, one year ahead returns, r^e_{t+1}, three year ahead returns, r^e_{t+3} and one year ahead unlevered returns, r^a_{t+1}. Across these various alternatives, there are significant differences between low and high MPK portfolios. The MPK-HML spread ranges from over 3.5% for unlevered within-industry returns to almost 11% for contemporaneous returns.

Next, Table 14 reports the results of portfolio sorts after controlling for firm size and book-to-market. To control for size, we allocate in each industry-year (so all sorts are industry-adjusted) by market capitalization. We then demean each firm's *mpk* by the mean of their industry-year-size group and sort firms into five portfolios based on this measure. We report the results in the top panel of Table 14. The table shows that even when controlling for size, high MPK firms tend to offer higher expected returns than low ones. We follow a similar procedure to control for book-to-market and report the results in the bottom panel of the table. Again, the spreads in expected returns remain after controlling for this variable.

As a second approach to controlling for these variables, Tables 15 and 16 display the results from double-sorting on MPK and size and book-to-market, respectively. To ensure that there are a sufficient number of firms in each portfolio, we use three portfolios along each dimension. The portfolios are ranked from low to high MPK along the columns and from small to large along the rows (so that, e.g., portfolio (1,1) contains small firms with low MPK and portfolio (3,1) large firms with low MPK). We calculate the MPK-HML spread as well as the small-minus-

Table 13: Excess Returns on MPK-Sorted Decile Portfolios

Portfolio	Low	2	3	4	5	6	7	8	9	High	MPK-HML
					Panel A: Not Industry-Adjusted						
r_t^e	5.71*	8.30**	8.96**	9.21***	9.86***	11.48***	11.45***	12.56***	14.09***	16.42***	10.72***
	(1.66)	(2.29)	(2.44)	(2.59)	(2.77)	(3.07)	(3.01)	(3.15)	(3.45)	(3.92)	(4.57)
r_{t+1}^e	6.89*	10.34***	11.59***	12.96***	13.17***	13.77***	13.95***	13.52***	13.14***	13.82***	6.95***
	(1.94)	(2.92)	(3.21)	(3.71)	(3.77)	(3.80)	(3.73)	(3.48)	(3.25)	(3.43)	(3.10)
r_{t+3}^e	8.01**	11.30***	12.46***	12.37***	12.53***	12.86***	14.05***	13.73***	12.97***	13.01***	5.00**
	(2.48)	(3.32)	(3.61)	(3.72)	(3.67)	(3.72)	(3.84)	(3.74)	(3.35)	(3.38)	(2.19)
r_{t+1}^a	3.61	5.66**	6.89***	8.17***	8.35***	9.02***	8.73***	8.59***	8.19***	8.25***	4.64***
	(1.40)	(2.31)	(2.76)	(3.34)	(3.46)	(3.56)	(3.35)	(3.14)	(2.89)	(3.11)	(3.07)
					Panel B: Industry-Adjusted						
r_t^e	6.88	7.16*	8.71**	9.12***	9.96***	11.22***	11.83***	12.72***	13.88***	17.71***	10.83***
	(1.45)	(1.83)	(2.40)	(2.62)	(2.83)	(3.23)	(3.27)	(3.30)	(3.41)	(3.98)	(8.04)
r_{t+1}^e	9.18*	13.00***	11.78***	11.31***	12.22***	13.20***	12.14***	13.27***	13.89***	13.48***	4.29***
	(1.95)	(3.33)	(3.32)	(3.34)	(3.65)	(3.80)	(3.42)	(3.54)	(3.53)	(3.16)	(3.24)
r_{t+3}^e	11.20**	12.68***	11.84***	12.68***	12.00***	12.08***	12.43***	12.76***	12.93***	14.81***	3.61***
	(2.50)	(3.47)	(3.53)	(3.85)	(3.72)	(3.72)	(3.62)	(3.51)	(3.43)	(3.68)	(2.68)
r_{t+1}^a	5.12	8.59***	7.21***	7.10***	7.67***	8.41***	7.72***	8.56***	9.00***	8.66***	3.53***
	(1.39)	(3.01)	(2.88)	(2.97)	(3.27)	(3.42)	(3.05)	(3.21)	(3.23)	(2.82)	(2.99)

Notes: This table reports stock market returns for portfolios sorted by *mpk*. r_t^e denotes equal-weighted contemporaneous annualized monthly excess stock returns (over the risk-free rate) measured in the year of the portfolio formation from January to December of year t. r_{t+1}^e denotes the analogous future returns, measured from July of year $t+1$ to June of year $t+2$. r_{t+3}^e denotes future returns further in the future, measured as returns from July of year $t+3$ to June of year $t+4$. r_{t+1}^a denotes equal-weighted unlevered ("asset") returns from from July of year $t+1$ to June of year $t+2$, where we use an unlimited liability model to unlever equity returns. Industry adjustment is done by de-meaning *mpk* by industry-year and sorting portfolios on de-meaned *mpk*, where industries are defined at the 4-digit SIC code level. t-statistics in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

big spread (the size premium). The left-hand panel reports unconditional expected returns and the right-hand panel after adjusting for industry. Reading across the rows, the table shows that within each size bin, high MPK firms tend to offer higher expected returns than low ones (although the spread is not always statistically significant, which may be a function of (a) either a small number of firms in some of the portfolios or (b) the fact that size and MPK tend to be correlated, e.g., Table 11). Table 16 reports analogous results using book-to-market, along with the high-minus-low spread (the value premium). Our findings are similar – high MPK firms offer higher expected returns than low ones. The MPK-HML spread is positive within each book-to-market bin and is generally large and statistically significant.

Table 14: Excess Returns on MPK-Sorted Portfolios Controlling for Size and Book-to-Market

Portfolio	Low	1	2	3	4	MPK-HML
			Panel A: Industry and Market Cap Adjusted			
r^e_{t+1}	12.18***	12.51***	12.95***	13.61***	14.70***	2.52**
	(2.60)	(3.29)	(3.67)	(3.56)	(3.40)	(2.12)
			Panel B: Industry and Book-to-Market Adjusted			
r^e_{t+1}	9.55**	10.44***	11.03***	12.77***	13.24***	3.70***
	(2.03)	(2.83)	(3.11)	(3.37)	(3.04)	(2.60)

Notes: This table reports stock market returns for portfolios sorted by *mpk*. Panel A contains *mpk*-sorted future excess returns after de-meaning by firms of similar market capitalization within the same industry. We split firms in each year-industry into three groups based on their market capitalization and then construct *mpk* residuals by subtracting the mean *mpk* of the industry-year-size group from firm *mpk*. We then sort firms into five portfolios based on their residuals. In Panel B we construct the analogue of this procedure using book-to-market ratios instead of market capitalization. We define an industry at the 4-digit SIC code level. We compute equal-weighted forward annualized monthly excess stock returns (over the risk-free rate), measured in the year following the portfolio formation, from July of year $t+1$ to June of year $t+2$. t-statistics in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 15: Excess Returns on MPK and Size Portfolios

Mkt Cap	MPK, Not Industry-Adjusted				MPK, Industry-Adjusted			
	1	2	3	HML	1	2	3	HML
1	15.41***	17.51***	16.75***	1.34	11.20**	12.31***	14.10***	2.90***
	(3.62)	(4.32)	(3.97)	(0.88)	(2.55)	(3.34)	(3.43)	(2.73)
2	8.26**	13.08***	11.76***	3.50**	13.39***	13.77***	14.13***	0.76
	(2.04)	(3.38)	(2.86)	(2.55)	(3.18)	(3.64)	(3.47)	(0.71)
3	8.39***	10.96***	9.74***	1.35	9.82***	11.01***	11.61***	1.79**
	(2.79)	(3.37)	(2.63)	(0.89)	(2.78)	(3.57)	(3.20)	(2.07)
Big-Small	-7.02**	-6.55***	-7.01***		-1.38	-1.30	-2.49*	
	(-2.50)	(-2.77)	(-2.90)		(-0.82)	(-0.88)	(-1.65)	

Notes: This table reports stock market returns for portfolios, double sorted by *mpk* and market capitalization. We measure equal-weighted forward annualized monthly excess stock returns (over the risk-free rate), measured in the year following the portfolio formation, from July of year $t+1$ to June of year $t+2$. t-statistics in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 16: Excess Returns on MPK and Book-to-Market Portfolios

B/M	MPK, Not Industry-Adjusted				MPK, Industry-Adjusted			
	1	2	3	HML	1	2	3	HML
1	4.60	9.42**	9.02**	4.43***	7.22*	9.88***	9.74**	2.52***
	(1.07)	(2.49)	(2.09)	(2.96)	(1.81)	(2.91)	(2.44)	(2.60)
2	9.98***	14.77***	14.64***	4.66***	9.79**	11.52***	12.82***	3.03***
	(2.98)	(4.24)	(3.87)	(3.23)	(2.47)	(3.42)	(3.28)	(3.14)
3	14.28***	16.66***	17.21***	2.93*	16.52***	15.60***	18.03***	1.51
	(4.48)	(4.57)	(4.43)	(1.85)	(4.14)	(4.31)	(4.59)	(1.59)
HML	9.68***	7.24***	8.19***		9.30***	5.72***	8.29***	
	(4.31)	(4.48)	(4.78)		(7.84)	(5.28)	(6.87)	

Notes: This table reports stock market returns for portfolios, double sorted by mpk and book-to-market ratios. We measure equal-weighted forward annualized monthly excess stock returns (over the risk-free rate), measured in the year following the portfolio formation, from July of year $t+1$ to June of year $t+2$. t-statistics in parentheses, computed using Newey-West standard errors. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$